

Data Visualization

460-4120

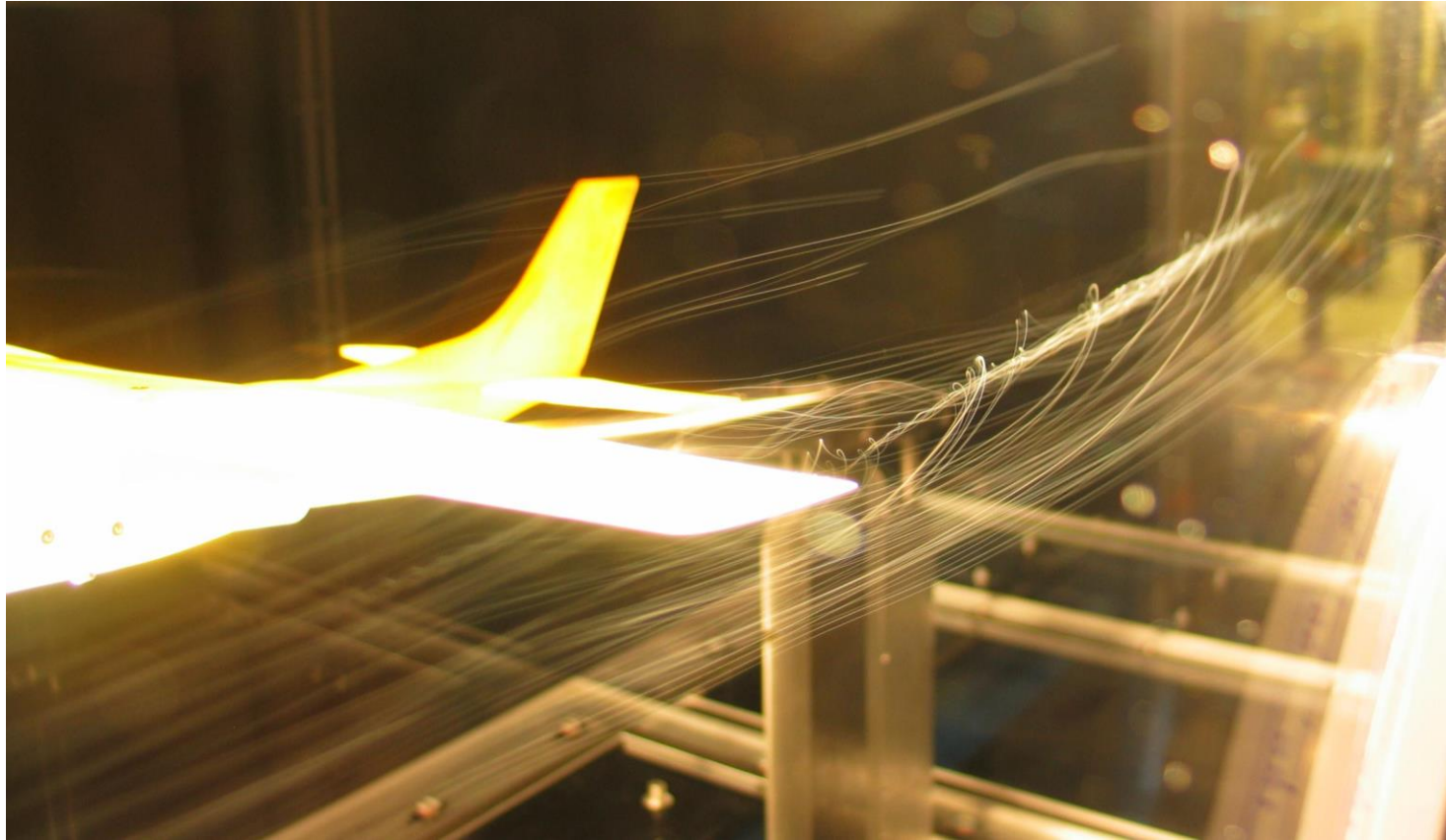
Fall 2024

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Vector Fields

- Vector field $\mathbf{v}: D \rightarrow \mathbb{R}^n$
- D is typically 2D planar surface or 2D surface embedded in 3D
- $n = 2$ fields tangent to 2D surface
- $n = 3$ volumetric fields
- When visualizing vector fields, we have to:
 - Map D to 2D screen (like with scalar fields)
 - Then we have 1 pixel for 2 or 3 scalar values (only 1 for scalar fields)

Vector Fields



A wind tunnel model of a Cessna 182 showing a wingtip vortex

Source: https://commons.wikimedia.org/wiki/File:Cessna_182_model-wingtip-vortex.jpg

Vector Fields

- We can compute derived scalar quantities from vector fields and use already known methods for these:
- **Divergence** ($\mathbb{R}^n \rightarrow \mathbb{R}$) degree of field's convergence or divergence

$$\operatorname{div} \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \text{ or equivalently } \operatorname{div} \mathbf{v} = \lim_{\Gamma \rightarrow 0} \frac{1}{|\Gamma|} \oint_{\Gamma} (\mathbf{v} \cdot \hat{\mathbf{n}}_{\Gamma}) ds \quad (\text{From a generalization of Stokes' Theorem})$$



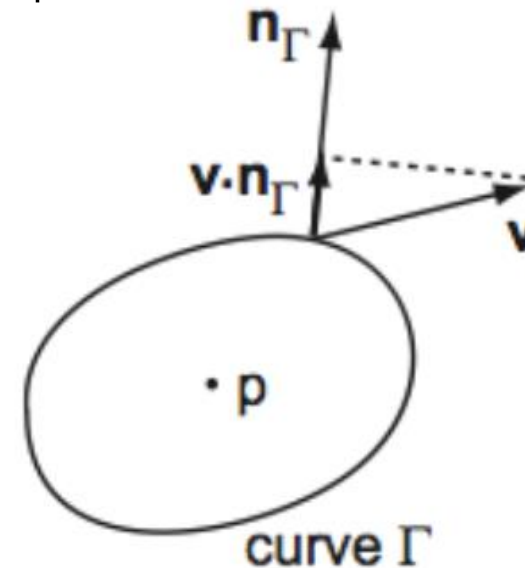
source
 $\operatorname{div} \mathbf{v} > 0$



sink
 $\operatorname{div} \mathbf{v} < 0$



laminar flow
 $\operatorname{div} \mathbf{v} = 0$

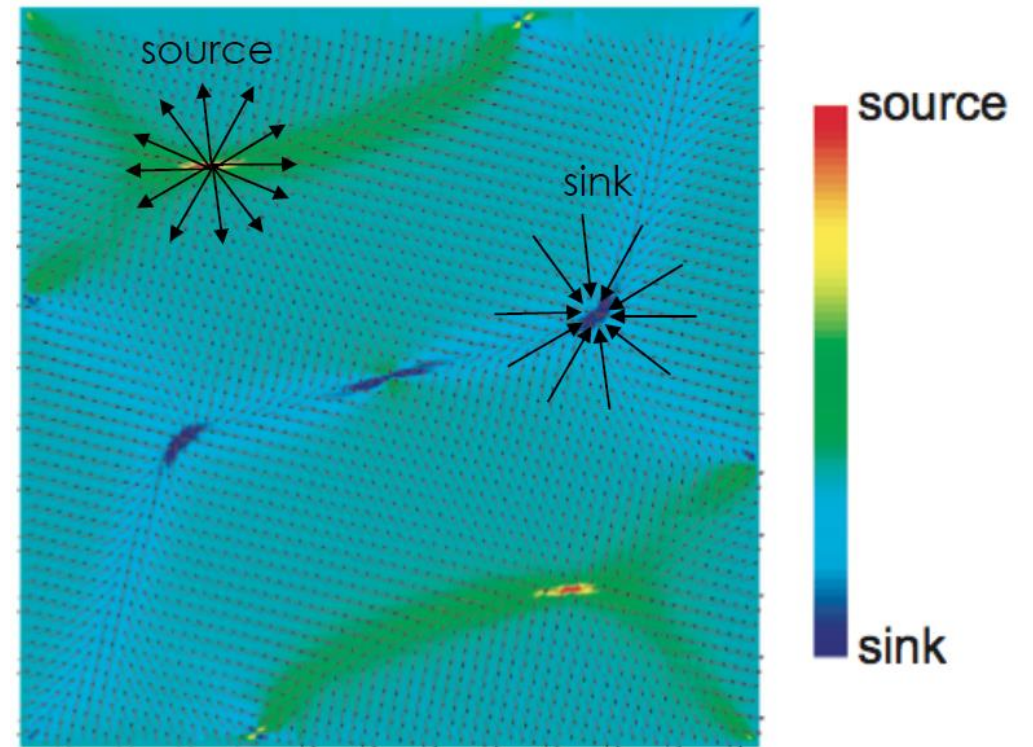


Problems With Differential Definition of Divergence

- A conventional centered difference always underestimate the true derivative

Vector Fields

- Divergence ($\mathbb{R}^n \rightarrow \mathbb{R}$)
- Informally, the divergence of a field is the difference between how much of the field enters and leaves a control volume
- In case of incompressible flow, the divergence of the velocity field is 0



Source: Eric Shaffer, *Vector Field Visualization*

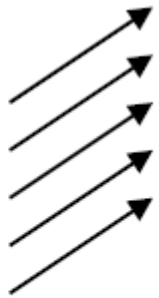
Vector Fields

- **Curl** (rotor, vorticity) ($\mathbb{R}^n \rightarrow \mathbb{R}^n$)

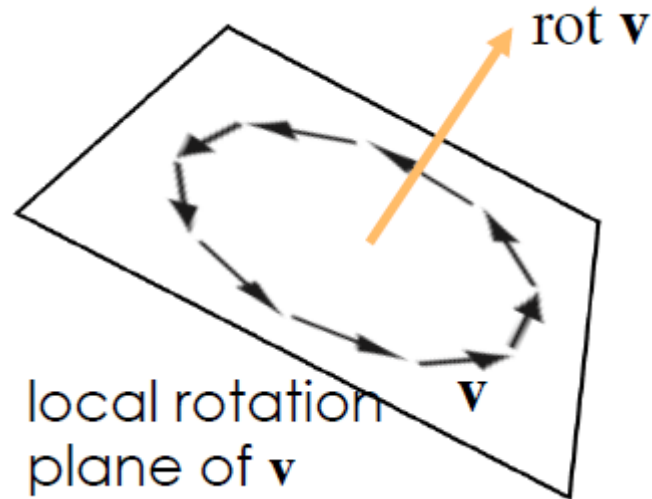
$$\text{rot } \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}, \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}, \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \text{ or } \text{rot } \mathbf{v} = \lim_{\Gamma \rightarrow 0} \frac{1}{|\Gamma|} \oint_{\Gamma} \mathbf{v} \cdot d\mathbf{s}$$



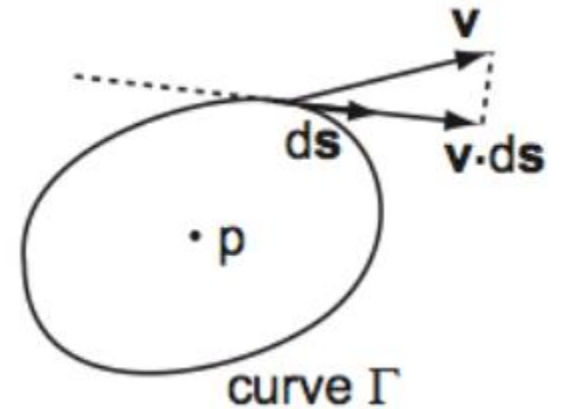
rotational flow
 $\|\text{rot } \mathbf{v}\| > 0$



laminar flow
 $\text{rot } \mathbf{v} = \mathbf{0}$



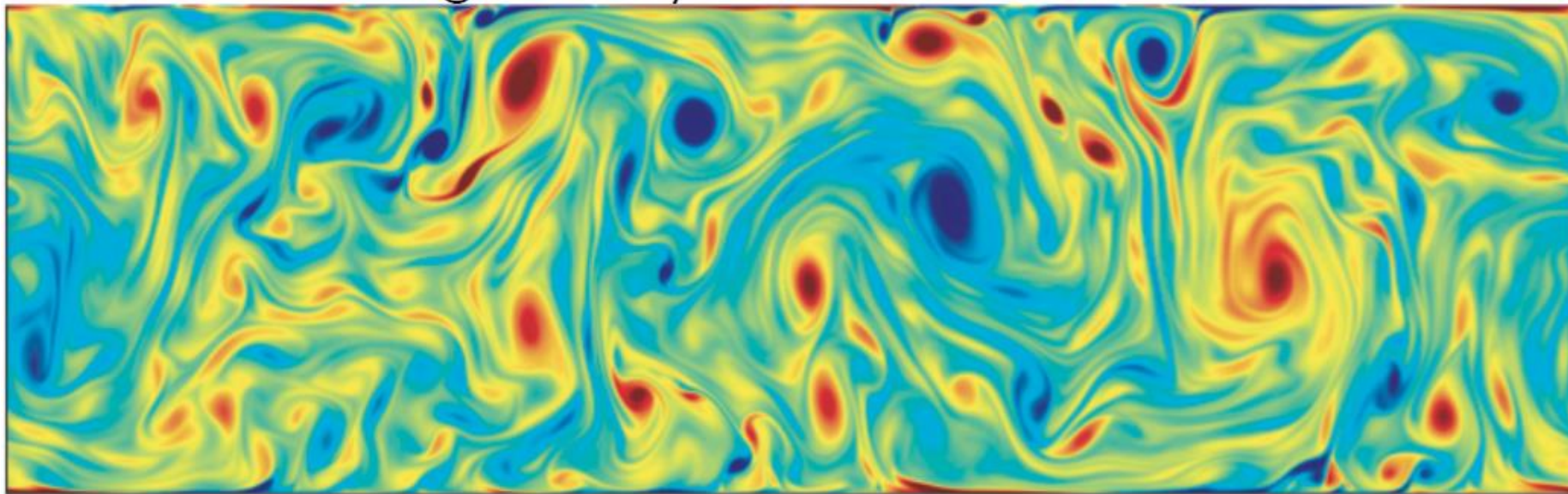
local rotation
plane of \mathbf{v}



- $\text{rot } \mathbf{v}$ is axis of the rotation and its magnitude is magnitude of rotation

Vector Fields

- Curl (rotor, vorticity) ($\mathbb{R}^n \rightarrow \mathbb{R}^n$)



counterclockwise laminar clockwise

- 2D fluid flow simulated by NSE and visualized using vorticity

Source: A Simple Fluid Solver based on the FFT, J. Stam, J. of Graphics Tools 6(2), 2001, 43-52

Approximating Derivatives

- Analytical functions
 - Ideally we are given a function in a closed form and we can (probably) compute previous derivatives analytically
 - Sometimes the given function is too complicated
 - We compute the derivatives numerically via differentiation
- Discretely sampled data
 - The best approach is to fit a function to the data and compute analytical derivatives
- Numerical evaluation may be faster than analytical

Approximating Derivatives

- Central-difference formulas of Order $O(h^2)$:

$$f'(x_0) \approx \frac{f_1 - f_{-1}}{2h}$$

$$f''(x_0) \approx \frac{f_1 - 2f_0 + f_{-1}}{h^2}$$

$$f^{(3)}(x_0) \approx \frac{f_2 - 2f_1 + 2f_{-1} - f_{-2}}{2h^3}$$

$$f^{(4)}(x_0) \approx \frac{f_2 - 4f_1 + 6f_0 - 4f_{-1} + f_{-2}}{h^4}$$

Approximating Derivatives

- Central-difference formulas of Order $O(h^4)$:

$$f'(x_0) \approx \frac{-f_2 + 8f_1 - 8f_{-1} + f_{-2}}{12h}$$

$$f''(x_0) \approx \frac{-f_2 + 16f_1 - 30f_0 + 16f_{-1} - f_{-2}}{12h^2}$$

$$f^{(3)}(x_0) \approx \frac{-f_3 + 8f_2 - 13f_1 + 13f_{-1} - 8f_{-2} + f_{-3}}{8h^3}$$

$$f^{(4)}(x_0) \approx \frac{-f_3 + 12f_2 - 39f_1 + 56f_0 - 39f_{-1} + 12f_{-2} - f_{-3}}{6h^4}$$

For an overview of how to approximate the derivative to any order of accuracy see https://en.wikipedia.org/wiki/Finite_difference_coefficient

Numerical Integration of ODE

Alternative notation of 1st order DE is

$$F(t, y(t), y'(t)) = 0,$$

where $y'(t)$ will explicitly appear in the eq. although t and $y(t)$ need not

- First-order differential equation (initial value problem)

$$y'(t) = f(t, y(t)), \quad y(t_0) = y_0$$

y ... unknown function

- We want to find an approximation of a nearby point on the trajectory y of an object placed in the vector field f by moving a short distance along a line tangent to the trajectory of this point
- We replace the derivative $y'(t)$ by the finite difference approximation (forward difference) $y'(t) \approx (y(t + \Delta t) - y(t))/\Delta t$ yielding

$$y(t + \Delta t) \approx y(t) + y'(t)\Delta t = y(t) + f(t, y(t))\Delta t$$

Numerical Integration of ODE

- First order Euler method:

$$\mathbf{x}(t + \Delta t) = \mathbf{x}(t) + \mathbf{v}(t, \mathbf{x}(t))\Delta t, \mathbf{x}(t_0) = \mathbf{x}_0$$

\mathbf{x} ... represents spatial position

\mathbf{v} ... vector field

Δt ... integration time step

\mathbf{x}_0 ... initial position

- Fast but not very accurate
- Higher methods available

Numerical Integration of ODE

- Second order Runge-Kutta method:

$$\mathbf{x}(t + \Delta t) = \mathbf{x}(t) + \frac{1}{2} (K_1 + K_2)$$

where

- $K_1 = \mathbf{v}(\mathbf{x}(t))\Delta t$
- $K_2 = \mathbf{v}(\mathbf{x}(t) + K_1)\Delta t$

Numerical Integration of ODE

- Fourth order Runge-Kutta method (aka RK4):

$$\mathbf{x}(t + \Delta t) = \mathbf{x}(t) + 1/6 (K_1 + 2K_2 + 2K_3 + K_4)$$

where

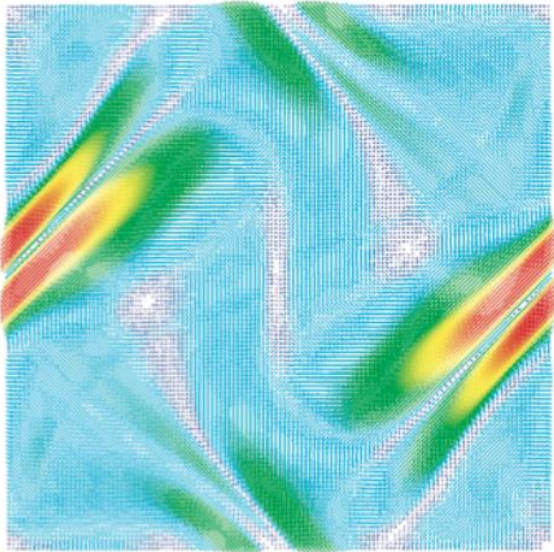
- $K_1 = \mathbf{v}(\mathbf{x}(t))\Delta t$
 - $K_2 = \mathbf{v}(\mathbf{x}(t) + 1/2 K_1)\Delta t$
 - $K_3 = \mathbf{v}(\mathbf{x}(t) + 1/2 K_2)\Delta t$
 - $K_4 = \mathbf{v}(\mathbf{x}(t) + K_3)\Delta t$
- There are many other methods (e.g. Verlet integration)

Glyphs

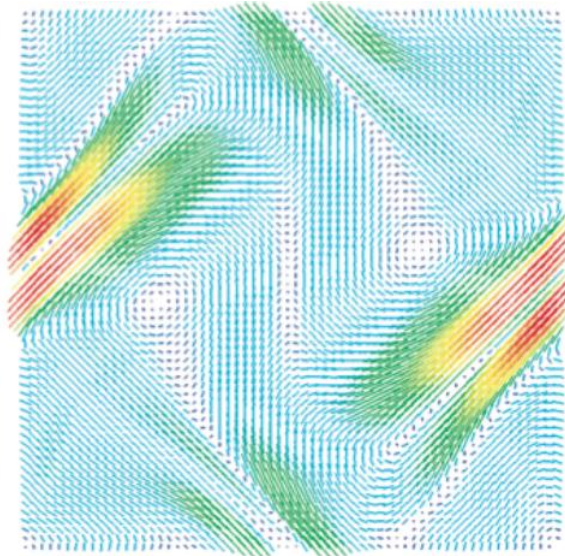
- Icons or signs for visualizing vector fields
 - Placed by (sub)sampling the dataset domain
 - Attributes (scale, color, orientation) map vector data at sample points
- Simplest glyph: Line segment (hedgehog plots)
 - Line from \boldsymbol{x} to $\boldsymbol{x} + k\boldsymbol{v}$
 - Optionally color map $|\boldsymbol{v}|$

Glyphs

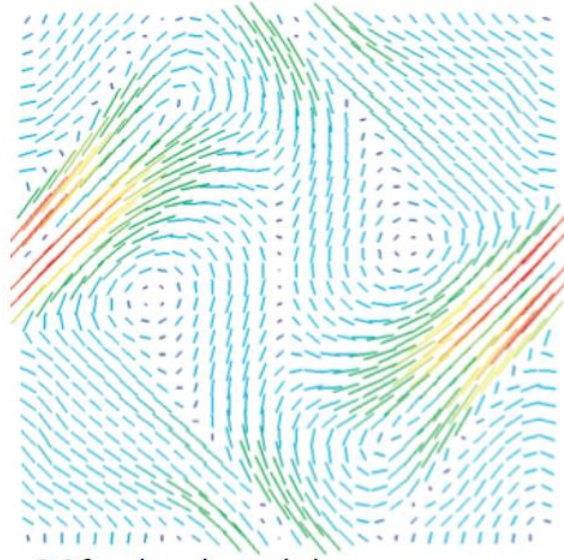
- Simplest glyph: Line segment (hedgehog plots)



128² glyph grid



64² glyph grid



32² glyph grid

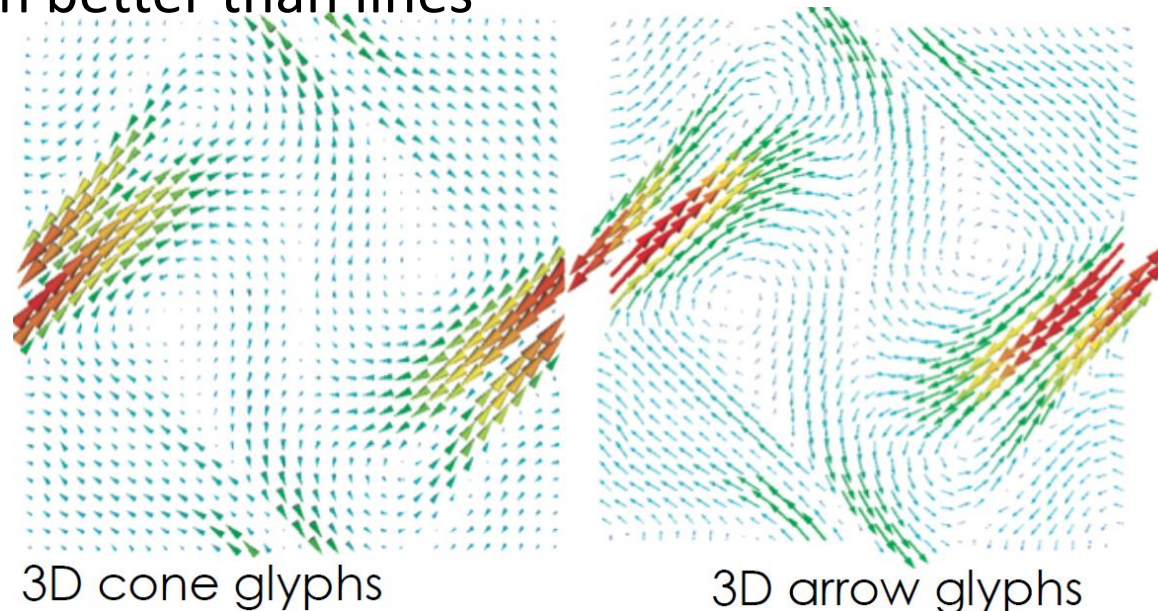


32² glyph grid, no line scaling

Source: Eric Shaffer, Vector Field Visualization

Glyphs

- Another variant: 3D cone and 3D arrow glyphs
- Shows orientation better than lines

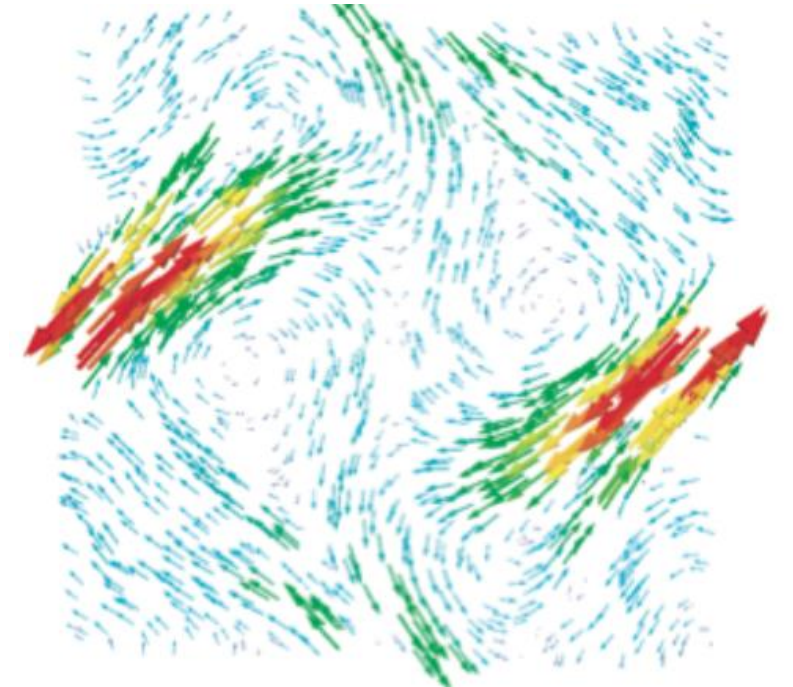


- Use shading to separate overlapping glyphs

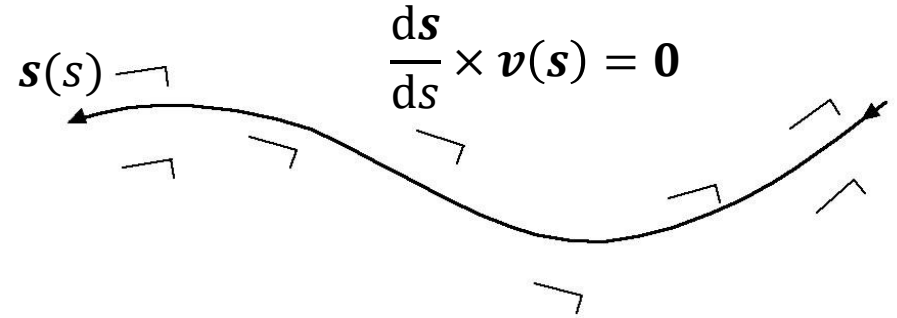
Source: Eric Shaffer, Vector Field Visualization

Glyphs Related Problems

- No interpolation in glyph space (unlike for scalar plots)
- A glyph take more space than a pixel (clutter)
- Complicated visual interpolation of arrows
- Scalar plots are dense, glyph plots are sparse
 - Glyph positioning is important:
 - Uniform grid
 - Rotated grid
 - Random samples (generally best solution)



Characteristic Lines

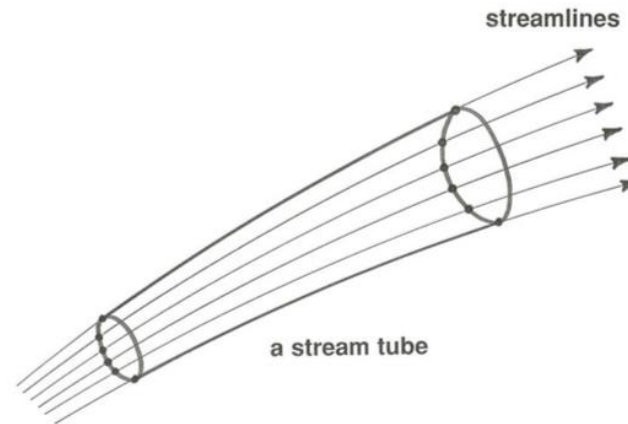


- Important approaches of characteristic lines in a vector field:
 - (1a) Stream line (proudnice) – curve everywhere tangential to the instantaneous vector (velocity) field (time independent vector field)
 - Local technique – initiated from one or a few particles
 - Trace of the direction of the flow at one single time step
 - Represents the direction of the flow at a given fixed time
 - Curves can look different at each time for unsteady flows
 - The scene becomes cluttered when the number of streamlines is increased
 - Computationally expensive

US Navy / The COMET Program

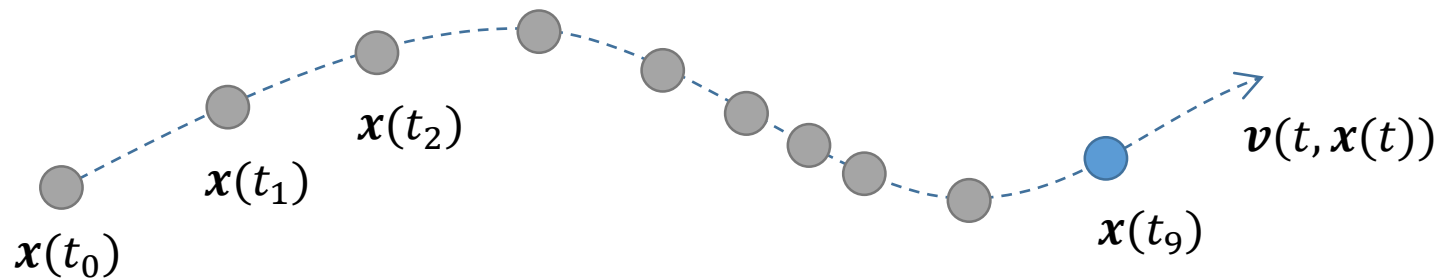
Characteristic Lines

- Important approaches of characteristic lines in a vector field:
 - (1b) Stream tubes – all stream lines of a stream together form the stream tube surface
 - There is flow through the tube but there is no flow either inward or outward across its surface



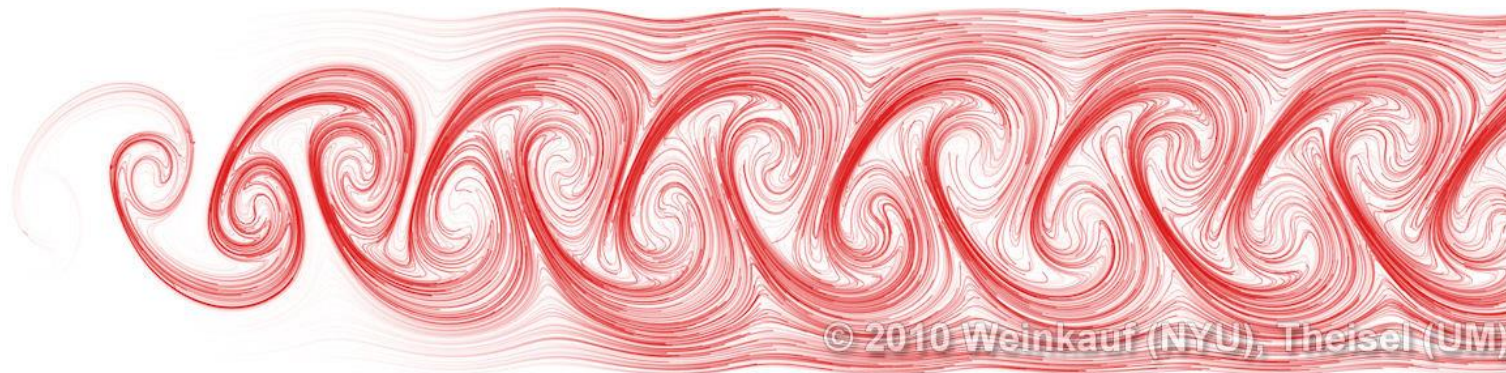
Characteristic Lines

- There are three extensions of stream lines for time-varying data:
 - (2) Path lines – trajectories of massless particles in the flow
 - Trace of a single particle in continuous time (at multiple snapshots in time)
 - The path line shows how the particle moves in the fluid



Characteristic Lines

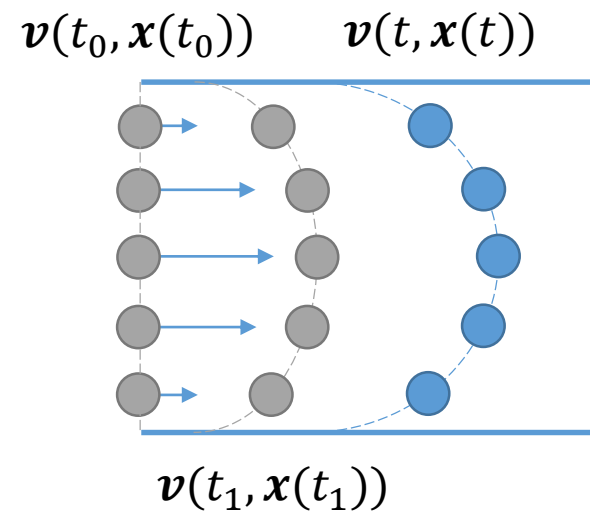
- (3) Streak lines – trace of dye that is released into the flow at a fixed position
 - Curve that connects the position of all the particles which are initiated at the certain point but at different times



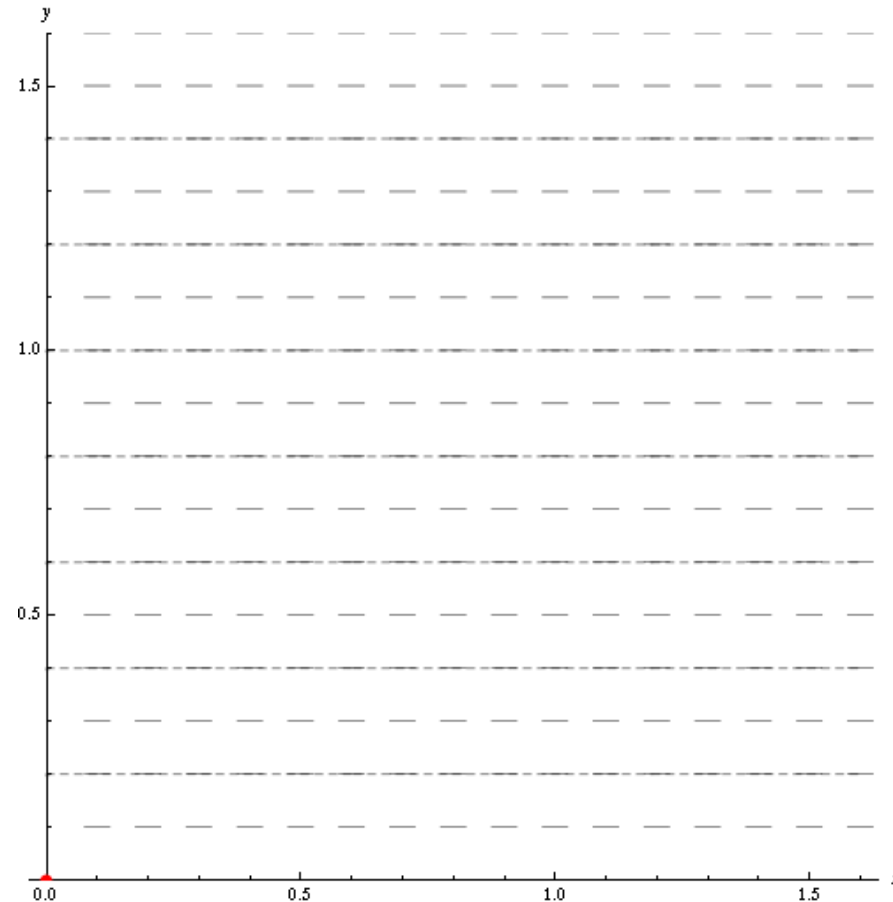
- In steady flow a path line = streak line = stream line

Characteristic Lines

- (3) Time lines – propagation of a line of massless particles in time



Characteristic Lines Example



The red particle moves in a flowing fluid; its *pathline* is traced in red; the tip of the trail of blue ink released from the origin follows the particle, but unlike the static pathline (which records the earlier motion of the dot), ink released after the red dot departs continues to move up with the flow. (This is a *streakline*.) The dashed lines represent contours of the velocity field (*streamlines*), showing the motion of the whole field at the same time.

Source:

https://en.wikipedia.org/wiki/Streamlines,_streaklines,_and_pathlines

Streamlines Example

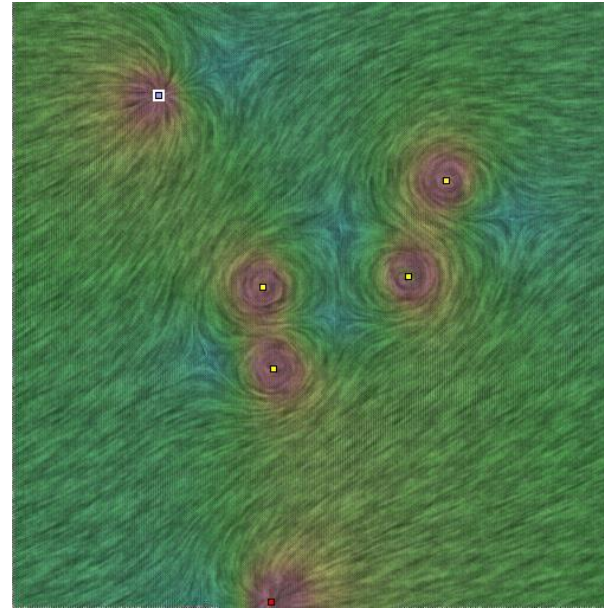
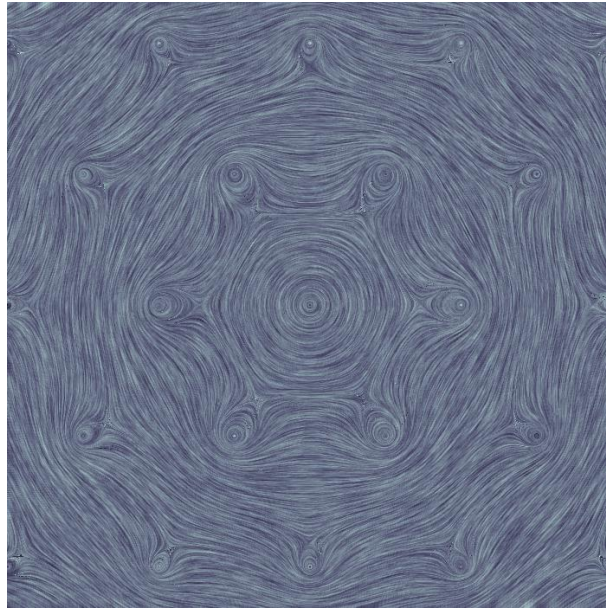


Magnetic fields shown as streamlines over a visible light and X-ray composite image of the M77 galaxy from the HST

Source:
<https://www.nasa.gov/image-feature/shaping-a-spiral-galaxy-0>

Line Integral Convolution

- LIC is a technique for representing two-dimensional vector fields
- The idea is to produce a texture which is highly correlated in the direction of the vector field but not correlated across the vector field
- The basic technique ignores both the magnitude of the vector field and its sign



Further Reading

- http://crcv.ucf.edu/projects/streakline_eccv
- <http://www.zhanpingliu.org/research/flowvis/lic/lic.htm>

Exercise

- Try to create the visualization of time-varying vector field representing fluid flow

- Hedgehogs show velocities of flow field at uniform grid vertices
- Background color reflects local velocities
- Euler ODE solver used for tracking of randomly scattered green particles
- Rotation of vector field shown by diverging color scheme

