

Image Features

In the previous exercise, we segmented image objects using the indexing algorithm. After this step, we have individual objects, and we would like to represent these objects with a few properties. These properties will be expressed as numbers. These numbers are called features. According to the features, the objects may be classified into classes. It is important to choose suitable features that will be capable to distinguish between the objects. The values of features should be similar for objects from the same class, and should be different between objects from different classes. For example, if we would like to distinguish between tomato and strawberries, we cannot use color as the feature, but the shape of the objects would be more useful. If we will have more than two classes of objects, we will usually need more than one feature to distinguish between the objects. Therefore, we will use a **feature vector**. In practice, there exist algorithms that use thousands of features, but they are not computed from white object areas as we have, but directly from the image data, i.e. edges, textures.

In the following subsections, we introduce some features that are frequently used in object classification, and that use only the information from the area, size, and shape, which is the case of our objects of single color.

Moments

The moments are frequently used as object features. The moment relative to the image coordinate system is defined as

$$m_{p,q} = \int \int x^p y^q f(x,y) dx dy, \quad (1)$$

where $f(x,y)$ is image function value, p and q set the degree of the moment (it should be non-negative value), the integral is on the image area of the recognized object. In the case of discrete images, Eq. (1) is defined as

$$m_{p,q} = \sum_x \sum_y x^p y^q f(x,y). \quad (2)$$

Since we are working with a binary segmented image (the pixel values are only 0 or 1), we can simplify the equation by omitting $f(x,y)$ - we work only with the pixels belonging to the object, i.e. $f(x,y) = 1$. If we choose the values $p = 0$ and $q = 0$, we get the moment $m_{0,0}$ that gives us the object area (the number of pixels belonging to the object). It can be used as a feature, but it should be noted that the values are different for the same class of objects of different scale.

By using moments $m_{1,0}$ and $m_{0,1}$, we can compute the center of mass of the object defined as

$$x_t = \frac{m_{1,0}}{m_{0,0}}, \quad y_t = \frac{m_{0,1}}{m_{0,0}}. \quad (3)$$

This value can be used as a feature itself, but, more frequently, it is used for computing the moments with respect to the axes passing through the center of mass. If we would use the moments from Eq. (2), the output value would depend on the location of the object within the image - the value will be different if an identical object is located in the top-left corner or in the bottom-right corner. It is due to the fact that we use the coordinates x and y . Therefore, we define the moment $\mu_{p,q}$ with respect to the axes passing through the center of mass as

$$\mu_{p,q} = \sum_x \sum_y (x - x_t)^p (y - y_t)^q f(x,y). \quad (4)$$

This moment is independent to the location of object within the object. In addition, we can normalize the moment $\mu_{p,q}$ and we get the moment v that is also independent to the object size. This moment is defined as

$$v_{p,q} = \frac{\mu_{p,q}}{(m_{0,0})^\gamma}, \quad \gamma = \frac{p+q}{2} + 1. \quad (5)$$

If we want a feature that is invariant to object rotation, it is necessary to compute the moments with respect to the axes of the object. These axes create a coordinate system with the beginning in the center of mass, and the axes are rotated by an angle Θ towards to the

image coordinate system. This rotation may be computed using the moments μ_{20} and μ_{02} and by constructing the covariance matrix in the form

$$\begin{bmatrix} \mu_{20} & \mu_{11} \\ \mu_{11} & \mu_{02} \end{bmatrix}. \quad (6)$$

The eigenvectors of this matrix correspond to the major and minor axis (the major axis is the eigenvector whose eigenvalue is higher), the angle Θ is computed as the angle between the eigenvector of the major axis and the major axis of image. The eigenvalues are computed as (the derivation is at the end of this text):

$$\lambda_1 = \frac{(\mu_{20} + \mu_{02}) + \sqrt{(\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2}}{2} \quad (7)$$

$$\lambda_2 = \frac{(\mu_{20} + \mu_{02}) - \sqrt{(\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2}}{2} \quad (8)$$

The ratio between these eigenvalues $\frac{\lambda_2}{\lambda_1}$ represents the ratio between the length of the minor and the major object axis that is invariant to object rotation (see Fig. 1).

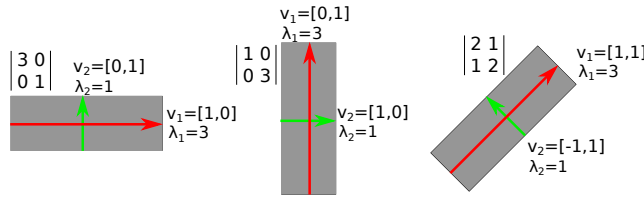


Figure 1: Illustration of eigenvectors and eigenvalues for features computation.

Relating circumference and area

Let P be a length of border of object (how to find out that a pixel is part of border? At least one of its neighbours is a pixel of background), and let A be an area of object (it can be computed as $m_{0,0}$). Kruhovost je definována vztahem

$$C = \frac{P^2}{A}. \quad (9)$$

For a square object, this value is 16 ($P = 4a$, $A = a * a$, where a is the length of one side), for a circle, it is 4π ($P = 2\pi r$, $A = \pi r^2$, where r is circle radius). It should be noted that these value are theoretical, since the objects will not exactly fit in discrete images.

Squareness

To compute this feature, we gradually rotate the object border (for example, with a rotation step 5°), and in each step, we find a rectangle with axes parallel to the image axes such that the object fits into the rectangle (programmatically, it is needed do find minimum and maximum of x and y for the rotated object). The rotation of object border points $[x, y]$ around its center of mass $[x_t, y_t]$ can be computed as

$$x_r = (x - x_t) \cos(\varphi) - (y - y_t) \sin(\varphi) + x_t, \quad (10)$$

$$y_r = (x - x_t) \sin(\varphi) + (y - y_t) \cos(\varphi) + y_t. \quad (11)$$

From all the rectangles, we choose the one with the lowest area. Let A be the area of object, let A_R be the area of the rectangle with the lowest area. The squareness is defined as

$$R = \frac{A}{A_R}. \quad (12)$$

As another feature, the ratio between the rectangle with the highest area (A_S) and the rectangle with the lowest area (A_R) can be used ($R_m = \frac{A_S}{A_R}$). For square, this value is around 0.5, the value 1 is for circle).

TASKS

Compute the features for the objects segmented in the image from the previous exercise. Use two features to describe objects in our images: ratio between circumference and area using Eq. (9), and the ratio between eigenvalues of covariance matrix, i.e.

$$F = \frac{\mu_{\min}}{\mu_{\max}} \quad (13)$$

where μ_{\min} and μ_{\max} are computed using Eq. (8) and Eq. (7), respectively. Based on the previous text, you can also use other features.

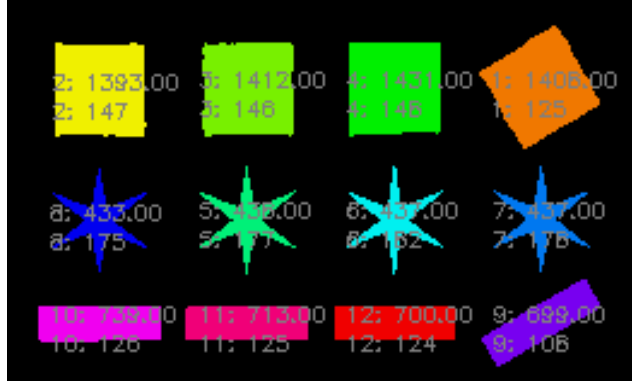


Figure 2: An example of computed areas and circumferences for each object. The first row in each object represents area, the second row represents circumference.

Eigenvalues of covariance matrix

Here we derive Eqs. (7) and (8).

$$\begin{aligned} & \begin{bmatrix} \mu_{20} & \mu_{11} \\ \mu_{11} & \mu_{02} \end{bmatrix} - \lambda \mathbf{I} = 0 \\ & \begin{bmatrix} \mu_{20} - \lambda & \mu_{11} \\ \mu_{11} & \mu_{02} - \lambda \end{bmatrix} = 0 \\ & (\mu_{20} - \lambda)(\mu_{02} - \lambda) - \mu_{11}^2 = 0 \\ & \lambda^2 - \lambda(\mu_{20} + \mu_{02}) + (\mu_{20}\mu_{02} - \mu_{11}^2) = 0 \end{aligned}$$

We compute the roots of quadratic equation

$$\begin{aligned} \lambda_{1,2} &= \frac{(\mu_{20} + \mu_{02}) \pm \sqrt{(\mu_{20} + \mu_{02})^2 - 4(\mu_{20}\mu_{02} - \mu_{11}^2)}}{2} = \\ &= \frac{(\mu_{20} + \mu_{02}) \pm \sqrt{\mu_{20}^2 + 2\mu_{20}\mu_{02} + \mu_{02}^2 - 4\mu_{20}\mu_{02} + 4\mu_{11}^2}}{2} = \\ &= \frac{(\mu_{20} + \mu_{02}) \pm \sqrt{\mu_{20}^2 - 2\mu_{20}\mu_{02} + \mu_{02}^2 + 4\mu_{11}^2}}{2} = \\ &= \frac{(\mu_{20} + \mu_{02}) \pm \sqrt{(\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2}}{2} \\ \lambda_1 &= \frac{(\mu_{20} + \mu_{02}) + \sqrt{(\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2}}{2} \\ \lambda_2 &= \frac{(\mu_{20} + \mu_{02}) - \sqrt{(\mu_{20} - \mu_{02})^2 + 4\mu_{11}^2}}{2} \end{aligned}$$