Perspective Transform

In this exercise, we'll implement a perspective transform to place one image into another one. The result is shown in Fig. 2.

A perspective transform is defined by the following equation

$$\begin{bmatrix} \phi(x,y) \\ \psi(x,y) \\ \omega(x,y) \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}, \tag{1}$$

where homogeneous coordinates after transform are $\phi(x,y)$, $\psi(x,y)$, and $\omega(x,y)$. The corresponding affine coordinates are then $\phi(x,y)/\omega(x,y)$ and $\psi(x,y)/\omega(x,y)$.

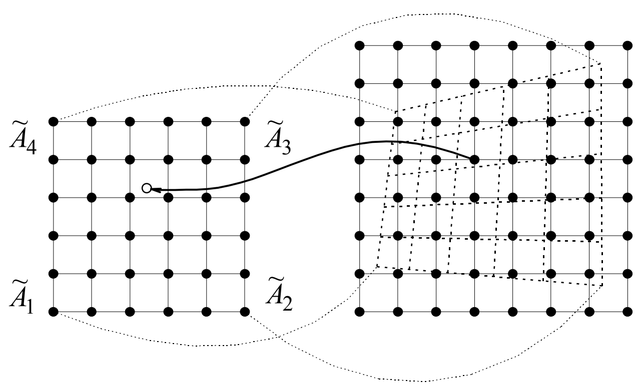

Figure 1: Projective transform of image.

Lets consider a projective transform described so that for four points $\tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \tilde{A}_4$ in an input image there are corresponding four points A_1, A_2, A_3, A_4 defined in an output image (in Fig. 1 points $\tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \tilde{A}_4$ are chosen as corner points). Lets illustrate the algorithm to derive the transform matrix from Eq. (1). Coordinates of point $\tilde{A}_i$ in the input image will be denoted as $\tilde{x}_i, \tilde{y}_i$. Location of a corresponding point A_i in the output image is described using x_i, y_i coordinates. Further, lets denote $\mathbf{p}_1 = (p_{11}, p_{12}, p_{13})^\top$, $\mathbf{p}_2 = (p_{21}, p_{22}, p_{23})^\top$, $\mathbf{p}_3 = (p_{31}, p_{32}, p_{33})^\top$, $\mathbf{x}_i = (x_i, y_i, 1)^\top$. Based on the Eq. (1),for each of the input points we obtain

$$\begin{bmatrix} \tilde{w}_i \tilde{x}_i \\ \tilde{w}_i \tilde{y}_i \\ \tilde{w}_i \end{bmatrix} = \begin{bmatrix} \mathbf{p}_1^\top \\ \mathbf{p}_2^\top \\ \mathbf{p}_3^\top \end{bmatrix} \mathbf{x}_i \,. \tag{2}$$

From the third equation in the formula (Eq. (2)) we get $\tilde{w}_i = \mathbf{p}_3^\top \mathbf{x}_i$. By substituting it into the first and second equation, we obtain

$$\begin{bmatrix} \mathbf{x}_i^\top & \mathbf{0} & -\tilde{x}_i \mathbf{x}_i^\top \\ \mathbf{0} & \mathbf{x}_i^\top & -\tilde{y}_i \mathbf{x}_i^\top \end{bmatrix} \begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \mathbf{p}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{3}$$

A complete version of Eq. (3) looks as the following

$$\begin{bmatrix} x_i & y_i & 1 & 0 & 0 & 0 & -\tilde{x}_i x_i & -\tilde{x}_i y_i & -\tilde{x}_i \\ 0 & 0 & 0 & x_i & y_i & 1 & -\tilde{y}_i x_i & -\tilde{y}_i y_i & -\tilde{y}_i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} . \tag{4}$$

At this point, we have 8 equations of 9 unknowns. To solve this issue, we can simply move the first column of the matrix in Eq. (4) to the right side of the equation. By doing so, we obtain the following system of equations

$$\begin{bmatrix} y_i & 1 & 0 & 0 & 0 & -\tilde{x}_i x_i & -\tilde{x}_i y_i & -\tilde{x}_i \\ 0 & 0 & x_i & y_i & 1 & -\tilde{y}_i x_i & -\tilde{y}_i y_i & -\tilde{y}_i \end{bmatrix} = \begin{bmatrix} -x_i \\ 0 \end{bmatrix} . \tag{5}$$

We still have 9 unknowns. The most easiest way to solve this system of equations is to set one of the coefficient of the perspective matrix to a fixed number, for example, $p_{11} = 1.0$ and compute the rest of coefficients using Eq. (5).

Implementation Details

To implement the perspective transform, you can use the following coordinates to define the correspondence between each point in the transform (staring from the top left corner in clockwise order):

Flag: $(0, 0)$, $(323, 0)$, $(323, 215)$, $(0, 215)$
Building: $(69, 107)$, $(227, 76)$, $(228, 122)$, $(66, 134)$

To solve system of equations, use OpenCV's `solve` function.

Expected Output

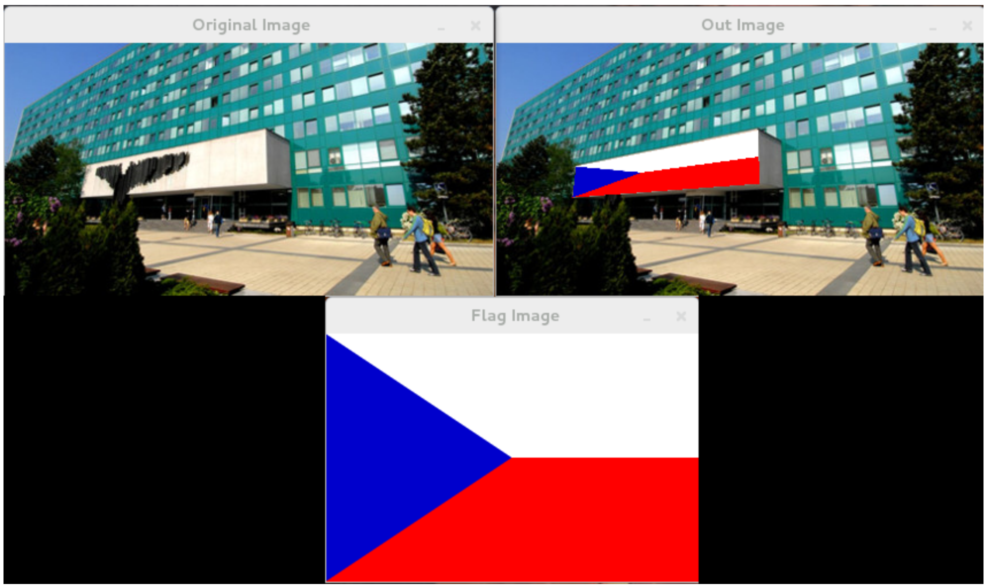

Figure 2: Input image (*top left* and *bottom* images); output image (*top right* image).