

Discrete Fourier Transform

Today's exercise is focused on implementation of the Discrete Fourier Transform (DFT). In the upcoming lecture, we'll implement inverse transform. The Fourier Transform computes frequency spectrum of the given input image f . This spectrum is denoted as F (it's a complex matrix with dimensions of the input image).

$$F(k, l) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m, n) \varphi_{k,l}(m, n). \quad (1)$$

The basis $\varphi_{k,l}$ is defined as

$$\varphi_{k,l}(m, n) = \frac{1}{\sqrt{MN}} e^{-i2\pi(\frac{mk}{M} + \frac{nl}{N})}, k = 0, 1, \dots, M-1 \text{ a } l = 0, 1, \dots, N-1. \quad (2)$$

To compute the basis, it's advantageous to use Eulers formula $e^{ix} = \cos(x) + i \sin(x)$. Thanks to this equation, the solution is split into real and imaginary parts. More precisely:

$$F(k, l) = R(k, l) + I(k, l). \quad (3)$$

The spectrum amplitude $|F(k, l)|$ is computed as follows

$$|F(k, l)| = \sqrt{R^2(k, l) + I^2(k, l)}. \quad (4)$$

The phase $\Phi(k, l)$ is defined as follows

$$\Phi(k, l) = \text{arctg} \left(\frac{I(k, l)}{R(k, l)} \right). \quad (5)$$

The power spectrum $P(k, l)$ may be computed easily as $P(k, l) = |F(k, l)|^2$.

You can display the power spectrum by logarithming the values of the spectrum and then normalizing values to the interval $\langle 0, 1 \rangle$. To see the power spectrum in a proper way it is good to swap first with third quadrant and also second with fourth quadrant. This swap should be performed on real and imaginary parts of the computed values. It will be useful later in filter application.

Hint: Use `double` data type to represent the input image, values of the frequency spectrum, and phase.

Expected Output

