Discrete Fourier Transform

Today's exercise is focused on implementation of the Discrete Fourier Transform (DFT). In the upcoming lecture, we'll implement inverse transform.
The Fourier Transform computes frequency spectrum of the given input image f. This spectrum is denoted as F (it's a complex matrix with dimensions of the input image).

$$F(k,l) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n) \varphi_{k,l}(m,n)\,. \tag{1}$$

The basis $\varphi_{k,l}$ is defined as

$$\varphi_{k,l}(m,n) = \frac{1}{\sqrt{MN}} e^{-i2\pi\left(\frac{mk}{M}+\frac{nl}{N}\right)}, k = 0,1,\ldots,M-1 \text{ a } l = 0,1,\ldots,N-1\,. \tag{2}$$

To compute the basis, it's advantageous to use Eulers formula $e^{ix} = \cos(x) + i\sin(x)$. Thanks to this equation, the solution is split into real and imaginary parts. More precisely:

$$F(k,l) = R(k,l) + I(k,l)\,. \tag{3}$$

The spectrum amplituda $|F(k,l)|$ is computed as follows

$$|F(k,l)| = \sqrt{R^2(k,l) + I^2(k,l)}\,. \tag{4}$$

The phase $\Phi(k,l)$ is defined as follows

$$\Phi(k,l) = \operatorname{arctg}\left(\frac{I(k,l)}{R(k,l)}\right)\,. \tag{5}$$

The power spectrum $P(k,l)$ may be computed easily as $P(k,l) = |F(k,l)|^2$.
You can display the power spectrum by logarithming the values of the spectrum and then normalizing values to the interval $\langle 0,1 \rangle$.

Hint: Use `double` data type to represent the input image, values of the frequency spectrum, and phase.

Expected Output