Notes on Calculating the Hamming Code

Layout of our test marker

	MSB 5	4	3	2	LSB 1
1	1	0	0	0	0
2	0	1	1	1	0
3	1	0	1	1	1
4	1	0	1	1	1
5	1	0	1	1	1

Flip all bits in the first column (we want to avoid empty rows)

	5	4	3	2	1
1	0	0	0	0	0
2	1	1	1	1	0
3	0	0	1	1	1
4	0	0	1	1	1
5	0	0	1	1	1

LSB … Least Significant Bit

MSB … Most Significant Bit

LSB				MSB	Notes
1	*2*	*3*	*4*	*5*	Column index
2^0	2^1	2^2	2^3	2^4	Power of two
P1	P2	D1	P3	D2	Positions of parity and data bits
1	**1**	**1**	**0**	**0**	Actual word from the 3rd row as an example (denoted as $\boldsymbol{a}_3$)
_	_	1	_	0	Data bits without parity bits
?	_	1	_	0	First parity bit: 1 + 0 is odd → P1 = 1
1	?	1	_	0	Second parity bit: 1 is odd → P2 = 1
1	1	1	?	0	Third parity bit: 0 is even → P3 = 0
1	**1**	**1**	**0**	**0**	Expected word (denoted as $\boldsymbol{b}_3$)

Words are the same and the Hamming distance $d_H(\boldsymbol{a}_3, \boldsymbol{b}_3) = 0$, see [1]. Now, we know that the 3rd row is a valid Hamming code. If that holds for all rows, we have found the valid marker. Note that our 5×5 field can represent up to $2^{2\cdot5}$ = 1024 values, i.e. we can generate 1024 different markers. For further reference, see [2].

[1] https://cs.wikipedia.org/wiki/Hammingova_vzd%C3%A1lenost

[2] https://users.cs.fiu.edu/~downeyt/cop3402/hamming.html