

## Light path from light source to camera

$$\text{normalize}(v) := \frac{v}{\|v\|}$$

$$n1 := 1.1111 \quad n2 := 1.0$$

$$n := \text{normalize} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \begin{array}{l} \text{surface} \\ \text{normal the} \\ \text{source} \end{array} \quad l := \text{normalize} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{bmatrix} 0.707 \\ 0.707 \\ 0 \end{bmatrix} \quad \begin{array}{l} \text{ray from} \\ \text{source toward} \\ \text{the surface} \end{array}$$

$$\cos_{\theta 1} := n \cdot l = 0.707 \quad \text{must be positive}$$

$$rr := l - (2 \cdot (n \cdot l)) \cdot n = \begin{bmatrix} 0.707 \\ -0.707 \\ 0 \end{bmatrix} \quad \text{reflected ray}$$

$$\cos_{\theta 2} := \sqrt{1 - \left(\frac{n2}{n1}\right)^2 \cdot (1 - (\cos_{\theta 1})^2)} = 0.771$$

$$rf := \frac{n2}{n1} \cdot l - \left(\frac{n2}{n1} \cdot \cos_{\theta 1} - \cos_{\theta 2}\right) \cdot n = \begin{bmatrix} 0.636 \\ 0.771 \\ 0 \end{bmatrix} \quad \text{refracted ray}$$

## Reverse light path from camera to light source

$$rd := -rf = \begin{bmatrix} -0.636 \\ -0.771 \\ 0 \end{bmatrix} \quad \begin{array}{l} \text{primary ray} \\ \text{i.e. -refracted ray} \end{array}$$

secondary ray B (i.e. reflected ray from source)

$$\cos_{\theta 2} := -n \cdot rd = 0.771$$

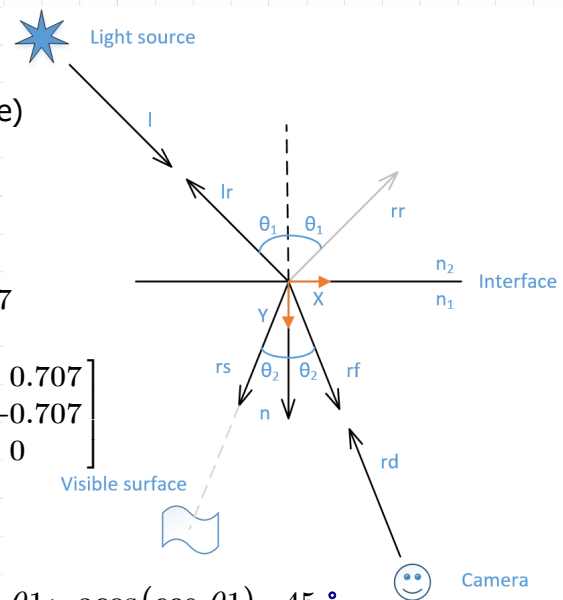
$$\cos_{\theta 1} := \sqrt{1 - \left(\frac{n1}{n2}\right)^2 \cdot (1 - (\cos_{\theta 2})^2)} = 0.707$$

$$rr := -\frac{n1}{n2} \cdot rd - \left(\frac{n1}{n2} \cdot \cos_{\theta 2} + \cos_{\theta 1}\right) \cdot n = \begin{bmatrix} 0.707 \\ -0.707 \\ 0 \end{bmatrix}$$

secondary ray A (i.e. -ray from source)

$$l := rr - (2 \cdot (n \cdot rr)) \cdot n = \begin{bmatrix} 0.707 \\ 0.707 \\ 0 \end{bmatrix}$$

$$lr := -l = \begin{bmatrix} -0.707 \\ -0.707 \\ 0 \end{bmatrix} \quad rs := rd - 2(n \cdot rd) \cdot n = \begin{bmatrix} -0.636 \\ 0.771 \\ 0 \end{bmatrix}$$



$$\theta 1 := \text{acos}(\cos_{\theta 1}) = 45^\circ$$

$$\theta 2 := \text{acos}(\cos_{\theta 2}) = 39.524^\circ$$

PG1, rev. 04