VSB TECHNICAL UNIVERSITY OF OSTRAVA | FACULTY OF ELECTRICAL ENGINEERING AND COMPUTER SCIENCE | DEPARTMENT OF COMPUTER SCIENCE

Computer Graphics I

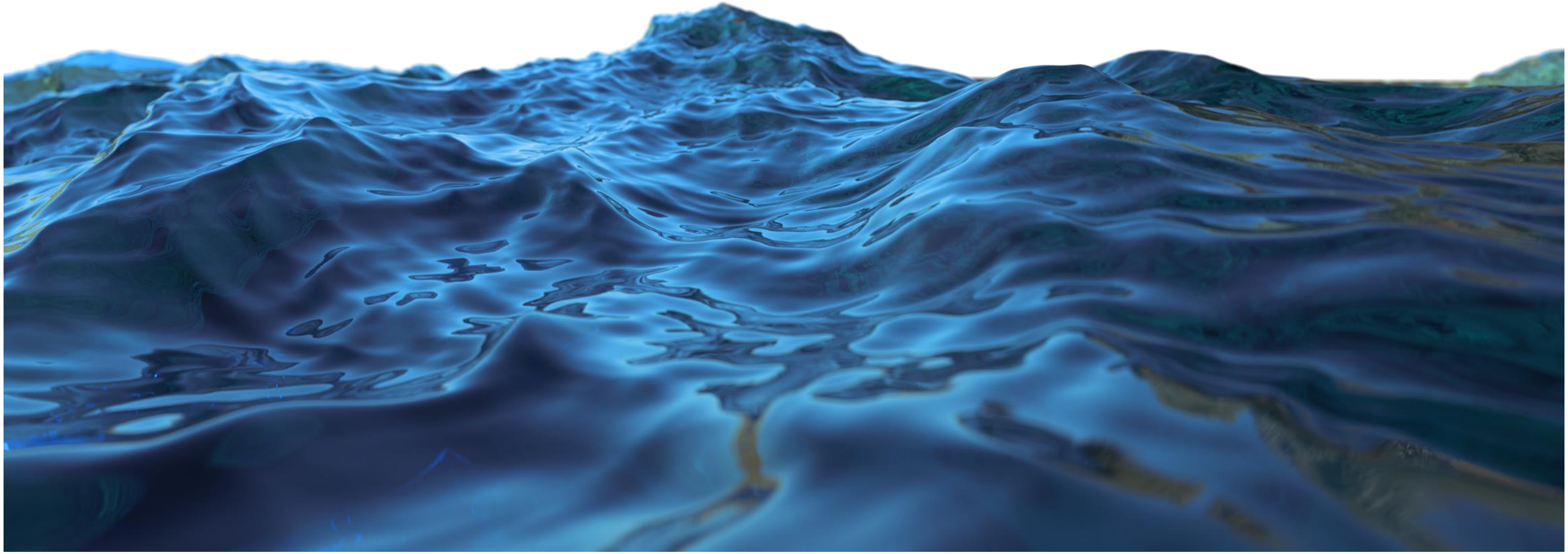
460-4078

Fall 2024

Last update 17. 11. 2024

Dielectric Transparent Materials

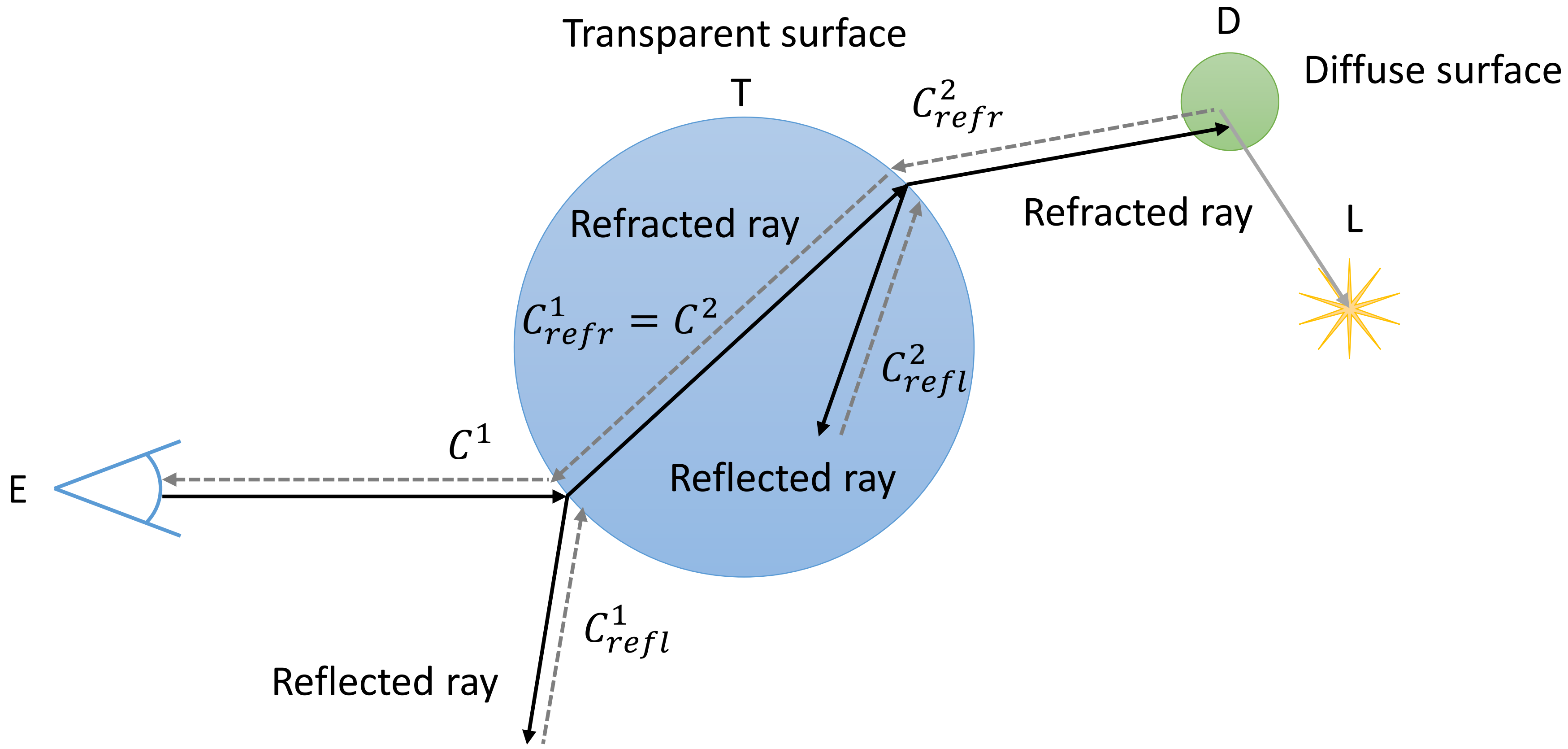
Water Surface



Dielectric Transparent Materials

- Two new rays are created at the intersection point:
  - Reflection ray
  - Refraction ray
- We get a tree of rays extending from the viewer
- Consequently, number of secondary rays grows (at worst) exponentially (2 → 4 → 8 → 16 …)
- Maximum depth parameter is even more important here

Dielectric Transparent Materials

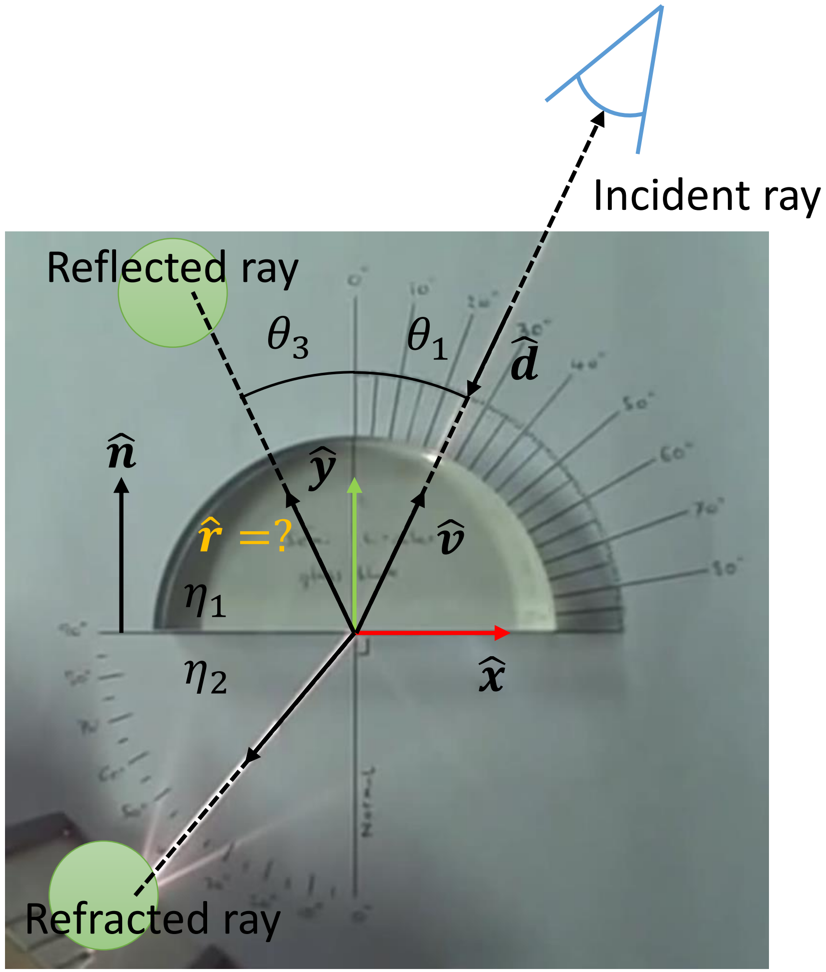


Geometry of Reflection

- Reflection of the incoming ray is govern by the law of reflection, i.e. the angle of incidence θ_1 is equal to the angle of reflection θ_3

$$\hat{\boldsymbol{r}} = 2(\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{n}})\, \hat{\boldsymbol{n}} - \hat{\boldsymbol{v}} =$$
$$= \hat{\boldsymbol{d}} - 2(\hat{\boldsymbol{d}} \cdot \hat{\boldsymbol{n}})\, \hat{\boldsymbol{n}}$$

Note that the orientation of $\hat{\boldsymbol{n}}$ is not important here

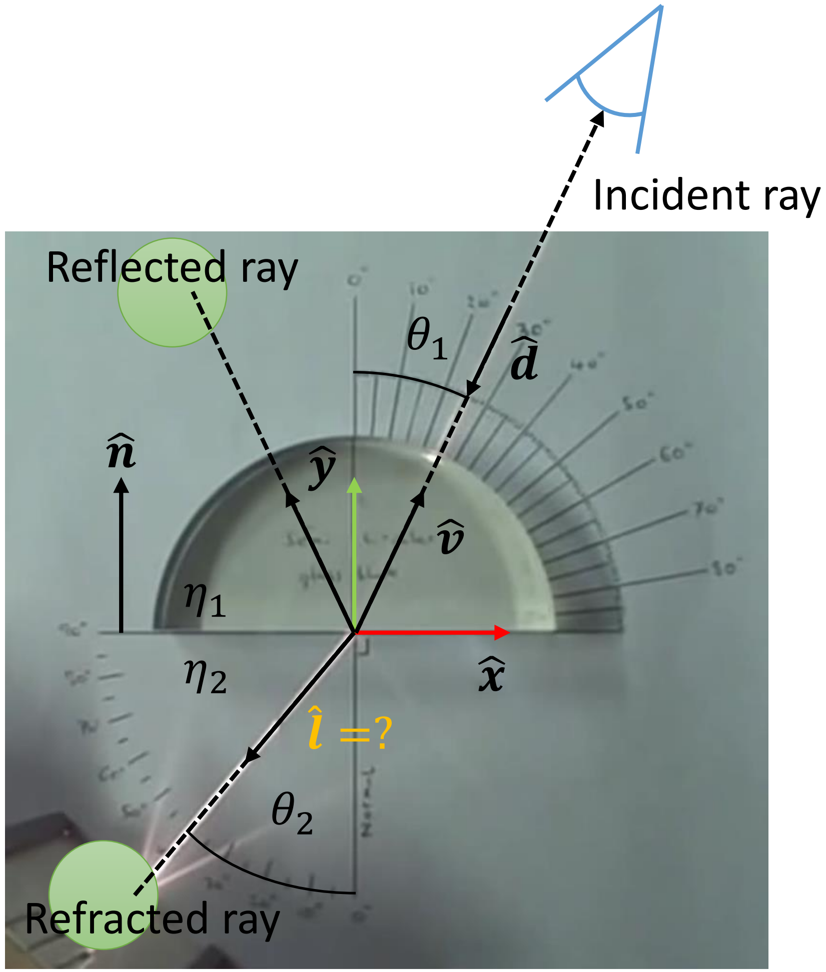


Geometry of Refraction

- Refraction is governed by Snell's law

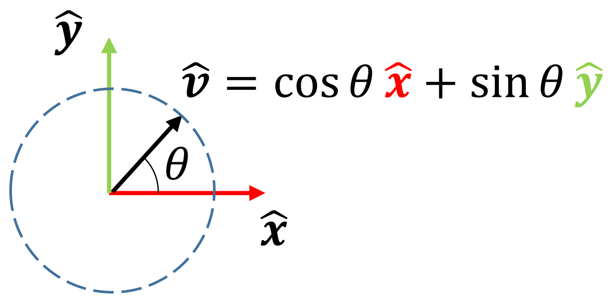
$$\eta_1 \sin\theta_1 = \eta_2 \sin\theta_2$$

- The question is how to compute the direction vector $\hat{\boldsymbol{l}}$ of the refracted ray based on the equation above
- What we know: refractive indices η_i of both materials or environments, normal vector $\hat{\boldsymbol{n}}$, and incoming direction $\hat{\boldsymbol{d}}$



Geometry of Refraction 1

Note that the plane of incidence is given by the normal and the direction of incoming ray

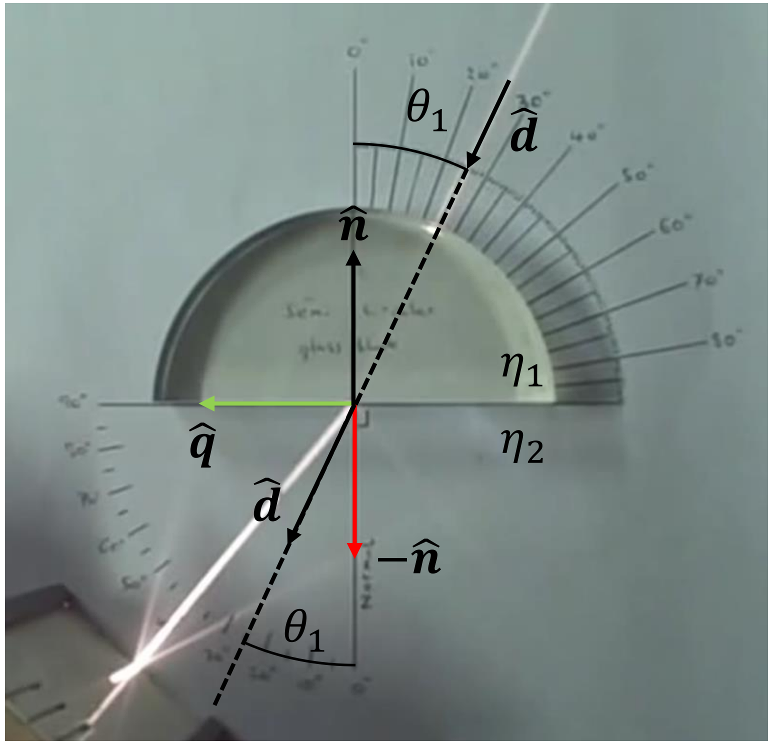


- We can introduce an auxiliary reference frame here
- Now we can express the known vector $\hat{\boldsymbol{d}}$ in terms of angle θ_1 and basis vectors $\hat{\boldsymbol{n}}$ and $\hat{\boldsymbol{q}}$ as follows

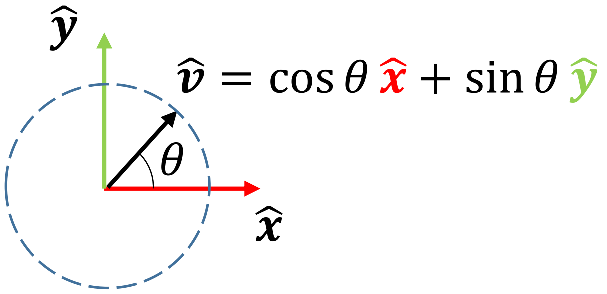
$$\hat{\boldsymbol{d}} = \sin\theta_1\,\hat{\boldsymbol{q}} - \cos\theta_1\,\hat{\boldsymbol{n}}$$

- We can rearrange the above formula and get the equation for unknown vector $\hat{\boldsymbol{q}}$

$$\hat{\boldsymbol{q}} = \left(\hat{\boldsymbol{d}} + \cos\theta_1\,\hat{\boldsymbol{n}}\right)/\sin\theta_1$$



Geometry of Refraction 2

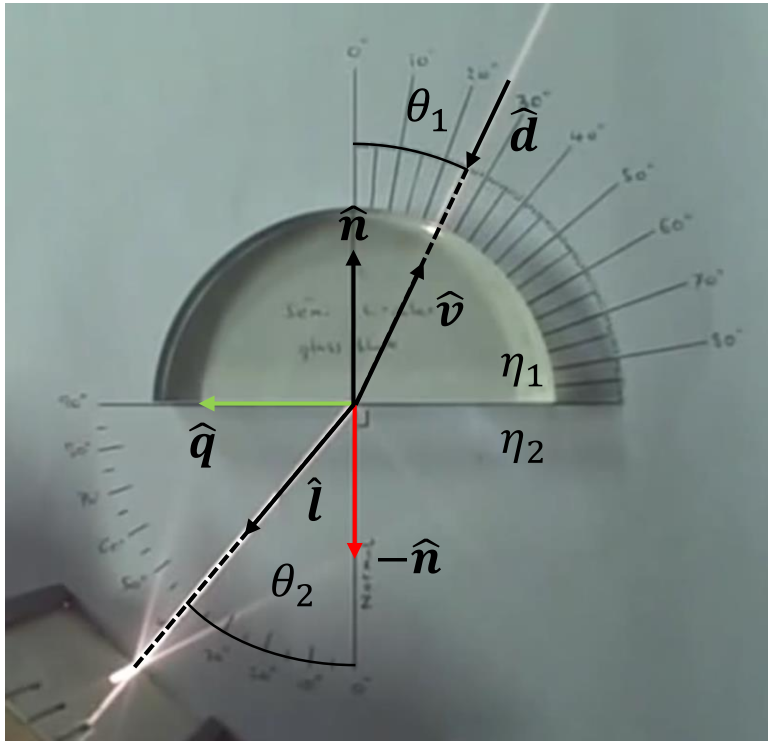


- We also know that the refracted direction $\hat{\boldsymbol{l}}$ can be expressed w.r.t. the reference frame $[-\hat{\boldsymbol{n}}, \hat{\boldsymbol{q}}]$ in the same way like vector $\hat{\boldsymbol{d}}$ as follows

$$\hat{\boldsymbol{l}} = \sin\theta_2\, \hat{\boldsymbol{q}} - \cos\theta_2\, \hat{\boldsymbol{n}}$$

- Other formulas needed to proceed further:

$$\cos\theta_1 = -\hat{\boldsymbol{d}} \cdot \hat{\boldsymbol{n}} = \hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{n}}$$
$$\sin^2\theta_1 = 1 - \cos^2\theta_1$$



Geometry of Refraction 3

- The final step is to put everything we know together …

$\eta_1 \sin\theta_1 = \eta_2 \sin\theta_2$

$\hat{\boldsymbol{l}} = \sin\theta_2\,\hat{\boldsymbol{q}} - \cos\theta_2\,\hat{\boldsymbol{n}} \qquad \hat{\boldsymbol{q}} = \left(\hat{\boldsymbol{d}} + \cos\theta_1\,\hat{\boldsymbol{n}}\right)/\sin\theta_1$

$\cos\theta_1 = -\hat{\boldsymbol{d}}\cdot\hat{\boldsymbol{n}} = \hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}} \qquad \cos^2\theta_2 = 1 - \sin^2\theta_2$

… and get the direction of refracted ray $\hat{\boldsymbol{l}}$ as follows

$$\hat{\boldsymbol{l}} = \frac{\sin\theta_2}{\sin\theta_1}\left(\hat{\boldsymbol{d}} + \cos\theta_1\,\hat{\boldsymbol{n}}\right) - \cos\theta_2\,\hat{\boldsymbol{n}} = \frac{\eta_1}{\eta_2}\left(\hat{\boldsymbol{d}} + \cos\theta_1\,\hat{\boldsymbol{n}}\right) - \cos\theta_2\,\hat{\boldsymbol{n}} =$$

$$= \frac{\eta_1}{\eta_2}\hat{\boldsymbol{d}} - \left(\frac{\eta_1}{\eta_2}\left(\hat{\boldsymbol{d}}\cdot\hat{\boldsymbol{n}}\right)\hat{\boldsymbol{n}}\right) - \sqrt{1-\sin^2\theta_2}\,\hat{\boldsymbol{n}} \quad \text{where}$$

$$\sin^2\theta_2 = \left(\frac{\eta_1}{\eta_2}\right)^2 \sin^2\theta_1 = \left(\frac{\eta_1}{\eta_2}\right)^2 \left(1-\cos^2\theta_1\right) = \left(\frac{\eta_1}{\eta_2}\right)^2 \left(1-\left(\hat{\boldsymbol{d}}\cdot\hat{\boldsymbol{n}}\right)^2\right)$$

Geometry of Refraction 4

- After some algebra we derive the final formula for the direction of the refracted ray…

$$\hat{\boldsymbol{l}} = \frac{\eta_1}{\eta_2}\hat{\boldsymbol{d}} - \left(\frac{\eta_1}{\eta_2}\left(\hat{\boldsymbol{d}} \cdot \hat{\boldsymbol{n}}\right) + \sqrt{1 - \left(\frac{\eta_1}{\eta_2}\right)^2 \left(1 - \left(\hat{\boldsymbol{d}} \cdot \hat{\boldsymbol{n}}\right)^2\right)}\right)\hat{\boldsymbol{n}}$$

Geometry of Reflection and Refraction

- Example with real numbers

$n1 = 1.5 \qquad n2 = 1$ indices of refraction

$d = \begin{bmatrix} -0.429 \\ -0.903 \\ 0 \end{bmatrix}$ incoming ray direction $\qquad n = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ surface normal

$v := -d \qquad v = \begin{bmatrix} 0.429 \\ 0.903 \\ 0 \end{bmatrix}$ direction towards the observer

$cos\_\theta 1 := n \cdot v = 0.903$ must be positive $\qquad \theta 1 := \mathrm{acos}(cos\_\theta 1) = 25.391\ deg$

$cos\_\theta 2 := \sqrt{1 - \left(\frac{n1}{n2}\right)^2 \cdot \left(1 - (cos\_\theta 1)^2\right)} = 0.766 \qquad \theta 2 := \mathrm{acos}(cos\_\theta 2) = 40.03\ deg$

direction of the refracted/transmitted ray

$l := \frac{n1}{n2} \cdot d + \left(\frac{n1}{n2} \cdot cos\_\theta 1 - cos\_\theta 2\right) \cdot n = \begin{bmatrix} -0.643 \\ -0.766 \\ 0 \end{bmatrix}$ check $\quad \mathrm{atan}\left(\frac{l_0}{l_1}\right) = 40.03\ deg$

$\sin(\theta 1) \cdot n1 = 0.643 \quad = \quad \sin(\theta 2) \cdot n2 = 0.643$

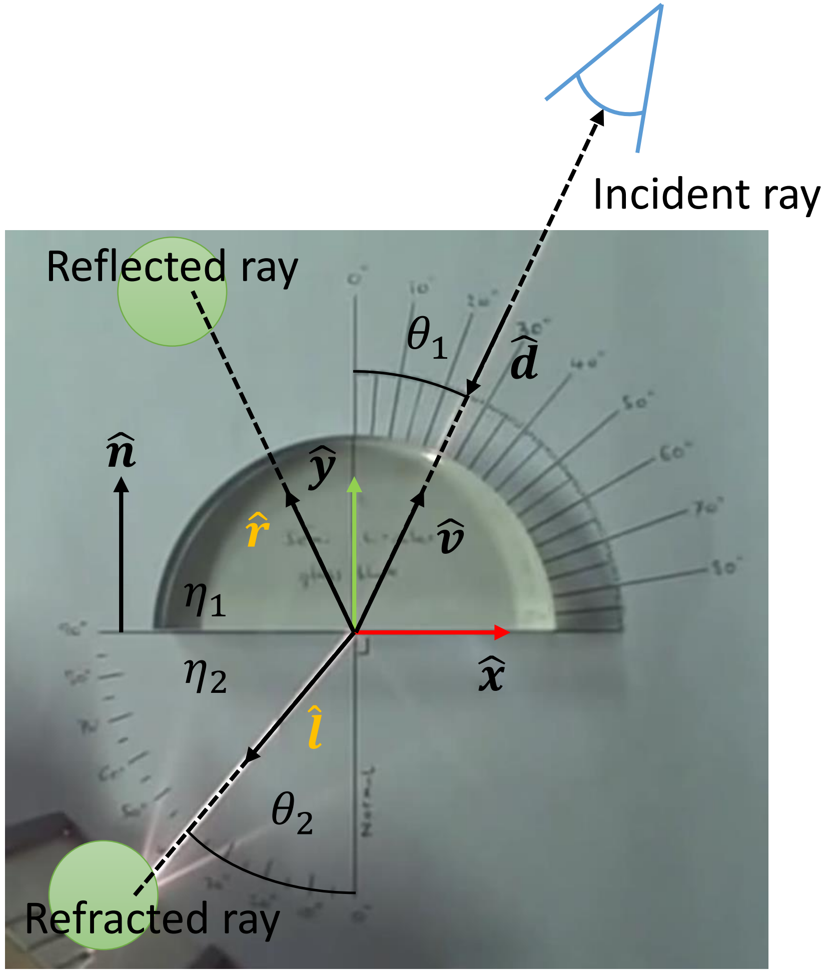
direction of the reflected ray

$r := (2 \cdot (v \cdot n)) \cdot n - v = \begin{bmatrix} -0.429 \\ 0.903 \\ 0 \end{bmatrix}$ check $\quad \mathrm{atan}\left(\frac{r_0}{r_1}\right) = -25.391\ deg$

PG1, rev. 07

derivation from the presentation results in an equivalent formultation

$l := \frac{n1}{n2} \cdot d - \left(\frac{n1}{n2} \cdot (d \cdot n) + \sqrt{1 - \left(\frac{n1}{n2}\right)^2 \cdot \left(1 - (d \cdot n)^2\right)}\right) n = \begin{bmatrix} -0.643 \\ -0.766 \\ 0 \end{bmatrix}$



Geometry of Reflection and Refraction

- What happens when we reverse the direction of the radiance (forward vs. backward ray tracing)?

$n1 := 1 \qquad n2 := 1.5$

$$v := \begin{bmatrix} -0.643 \\ -0.766 \\ 0 \end{bmatrix} \qquad d := -v = \begin{bmatrix} 0.643 \\ 0.766 \\ 0 \end{bmatrix} \qquad n := \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$cos\_\theta 1 := n \cdot v = 0.766 \qquad \theta 1 := \mathrm{acos}(cos\_\theta 1) = 40.004\ deg$$

$$cos\_\theta 2 := \sqrt{1 - \left(\frac{n1}{n2}\right)^2 \cdot \left(1 - (cos\_\theta 1)^2\right)} = 0.904 \qquad \theta 2 := \mathrm{acos}(cos\_\theta 2) = 25.376\ deg$$

$$l := \frac{n1}{n2} \cdot d + \left(\frac{n1}{n2} \cdot cos\_\theta 1 - cos\_\theta 2\right) \cdot n = \begin{bmatrix} 0.429 \\ 0.904 \\ 0 \end{bmatrix} \qquad \text{check} \quad \mathrm{atan}\left(\frac{l_0}{l_1}\right) = 25.382\ deg$$

$$\sin(\theta 1) \cdot n1 = 0.643 \quad = \quad \sin(\theta 2) \cdot n2 = 0.643$$

$$r := (2 \cdot (v \cdot n)) \cdot n - v = \begin{bmatrix} 0.643 \\ -0.766 \\ 0 \end{bmatrix} \qquad \text{check} \quad \mathrm{atan}\left(\frac{r_0}{r_1}\right) = -40.011\ deg$$

equivalent formulas

$$l := \frac{n1}{n2} \cdot d - \left(\frac{n1}{n2} \cdot (d \cdot n) + \sqrt{1 - \left(\frac{n1}{n2}\right)^2 \cdot \left(1 - (d \cdot n)^2\right)}\right) n = \begin{bmatrix} 0.429 \\ 0.904 \\ 0 \end{bmatrix}$$

$$l := \frac{n1}{n2} \cdot d + \left(\frac{n1}{n2} \cdot (-d \cdot n) - \sqrt{1 - \left(\frac{n1}{n2}\right)^2 \cdot \left(1 - (-d \cdot n)^2\right)}\right) n = \begin{bmatrix} 0.429 \\ 0.904 \\ 0 \end{bmatrix}$$

Conclusion from the experiment: it doesn't matter

More Efficient Method for Refraction Vector

$n1 := 1.5$ $\quad n2 := 1.0$ $\quad m := \frac{n1}{n2} = 1.5$

$d := \begin{bmatrix} -0.429 \\ -0.903 \\ 0 \end{bmatrix}$ $\quad n := \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$v := -d = \begin{bmatrix} 0.429 \\ 0.903 \\ 0 \end{bmatrix}$

Bec's method for efficient computing the refraction vector

$cos\_\theta1 := v \cdot n = 0.903$ $\quad \theta1 := \operatorname{acos}(cos\_\theta1) = 25.445\ \boldsymbol{deg}$

$w := m \cdot cos\_\theta1 = 1.355$

$k := \sqrt{1 + (w - m) \cdot (w + m)} = 0.765$

$l := (w - k) \cdot n - m \cdot v = \begin{bmatrix} -0.644 \\ -0.765 \\ 0 \end{bmatrix}$

Oliveira's approximate method

$c := 1.0$ (c is somewhere around 1.0 for simulating water)

$l := \frac{-c \cdot n - v}{\lVert -c \cdot n - v \rVert} = \begin{bmatrix} -0.22 \\ -0.976 \\ 0 \end{bmatrix}$

Amount of Refraction and Reflection

- Governed by Fresnel equations

$$Rs := \left(\frac{n2 \cdot cos\_\theta 2 - n1 \cdot cos\_\theta 1}{n2 \cdot cos\_\theta 2 + n1 \cdot cos\_\theta 1}\right)^2 = 0.077$$

$$Rp := \left(\frac{n2 \cdot cos\_\theta 1 - n1 \cdot cos\_\theta 2}{n2 \cdot cos\_\theta 1 + n1 \cdot cos\_\theta 2}\right)^2 = 0.014$$

$$R := \frac{Rs + Rp}{2} = 0.046 \qquad \text{coef. of reflected ray}$$

$$T := 1 - R = 0.954 \qquad \text{coef. of refracted/transmited ray}$$

Amount of Refraction and Reflection

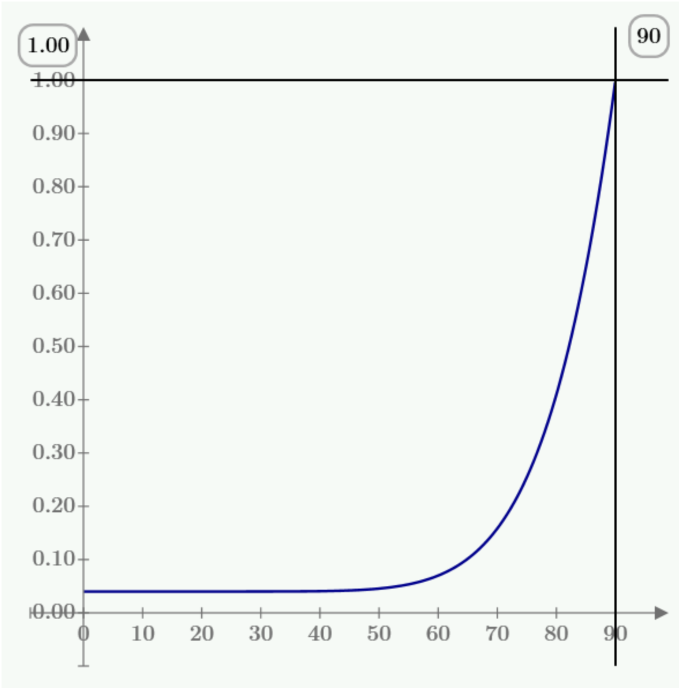
- Reflection coefficient R can be approximated by Schlick's model

$$R(\theta) \approx F_0 + (1 - F_0)(1 - \cos\theta)^5$$

where $F_0 = \left(\frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}\right)^2$ and

$$\theta = \begin{cases} \theta_i & \text{if } \eta_1 \leq \eta_2 \\ \theta_t & \text{otherwise} \end{cases}$$

Alternative approx. formula $R(\theta) \approx F_0 + (1 - F_0)\, 2^{(-5.55473\cos(\theta) - 6.98316)\cos(\theta)}$



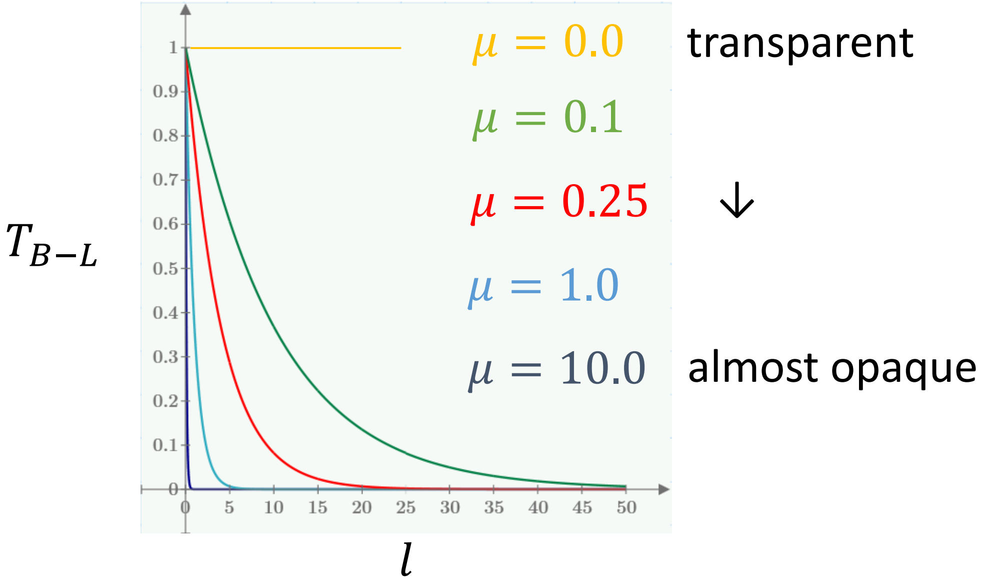
- Also note that when TIR occurs we should return 1
- The approximation fails if η_2 increases (e.g. $\eta_1 = 1$ and $\eta_2 = 5$)

Attenuation of Refracted Ray

- Beer–Lambert law relates the attenuation of light to the properties of the material through which the light is travelling
- Relation for the transmittance in case of uniform attenuation

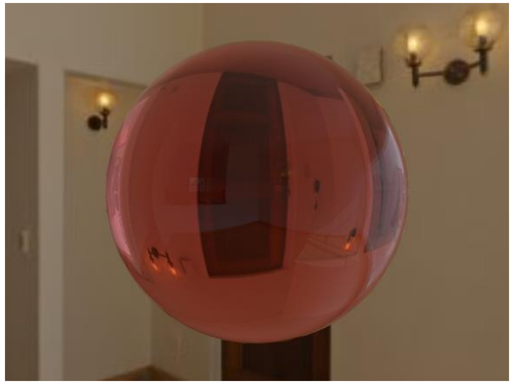
$$T_{B-L}(l) = e^{-\mu l}$$

- μ … attenuation coefficient $\langle 0, \infty)$
- l … length of the ray segment through the material sample



Attenuation of Refracted Ray in Glass ($\eta = 1.5$)

$\mu = (0.1, 1.0, 1.0)$



$\mu = (1.0, 0.1, 1.0)$

$\mu = (1.0, 1.0, 0.1)$

$\mu = (0.1, 0.1, 0.1)$

$\mu = (0.5, 0.5, 0.5)$

$\mu = (1.0, 1.0, 1.0)$

$$C = \left[C_{refl}\, R + C_{refr}(1 - R)\right] T_{B-L}((\eta_1 \text{ is air})?\ l = 0 : l = t_{hit})$$

Background (Environment) Images

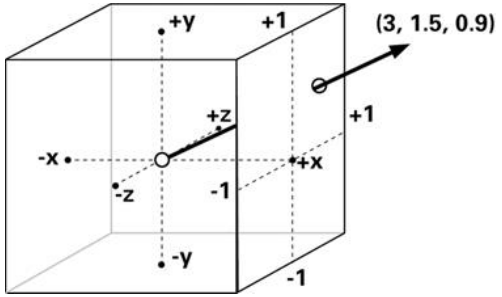
- Spherical map – one spherical texture

```
class SphericalMap
{
public:
  SphericalMap( const std::string & file_name );

  Color3f texel( const float x, const float y, const float z ) const;

private:
  std::unique_ptr<Texture> texture_;
};

SphericalMap::SphericalMap( const std::string & file_name )
{
  // TODO 1 load spherical texture from file_name and store it in texture_
}

Color3f SphericalMap::texel( const float x, const float y, const float z ) const
{
  // TODO 2 compute (u, v) coordinates from direction (x, y, z) using spherical mapping
  // TODO 3 return bilinear interpolation of a texel at (u, v)
}
```

Source: https://hdrihaven.com

Background (Environment) Images

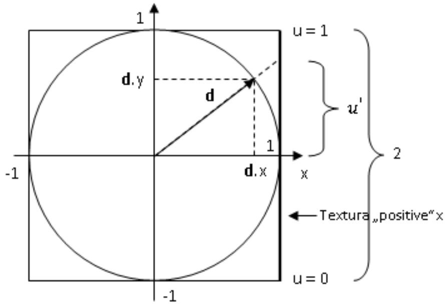
- Cube map – six square textures $\{\pm x, \pm y, \pm z\}$
- Firstly, select the one of the six maps



Based on the index of largest component in absolute value of directional vector $\widehat{\boldsymbol{d}}$ select the one of the six square maps.

Source: http://www.humus.name

- Secondly, compute u and v coordinates



From similar triangles we can easily see that $\frac{u'}{1} = \frac{d.y}{d.x}$ and after the normalization of u' we get $u = \frac{u'+1}{2}$. The same holds for v coordinate.

Textures Bilinear Interpolation

- Colors (or other values) obtained from textures should be interpolated using bilinear interpolation at least

Nearest neighbor interpolation

1 spp at pixel center

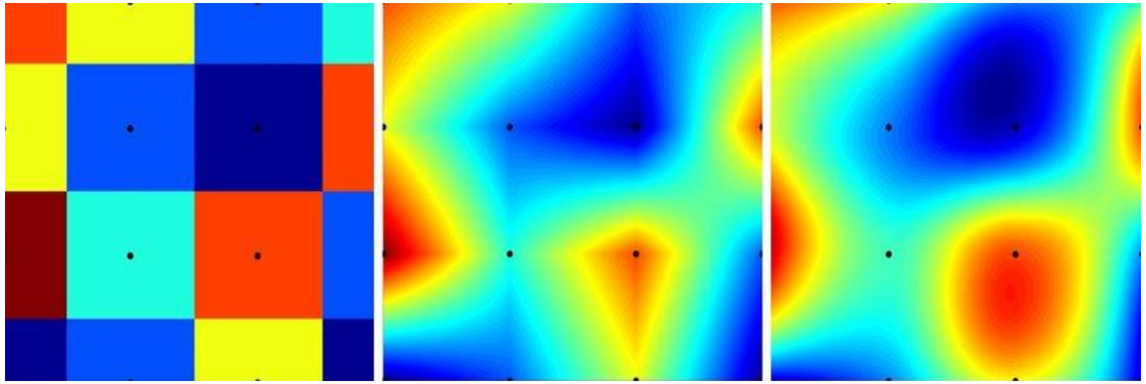
1 random spp

Bilinear interpolation

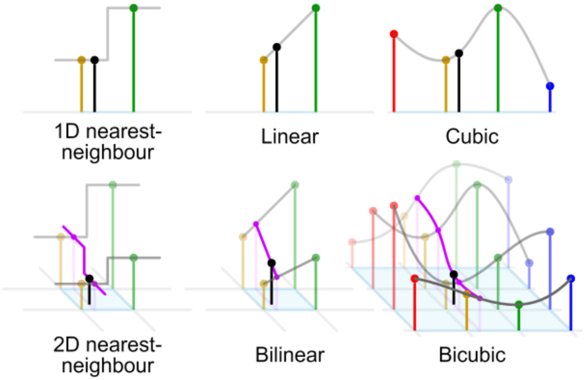
1 spp at pixel center

Bilinear Interpolation

- Number of required samples
  - Nearest neighbor: 1 sample
  - 2D bilinear interpolation: 2×2=4 samples
  - 2D bicubic interpolation: 4×4=16 samples



Nearest neighbor　　Bilinear　　Bicubic



Source: https://en.wikipedia.org/wiki/Bilinear_interpolation

Bilinear Interpolation

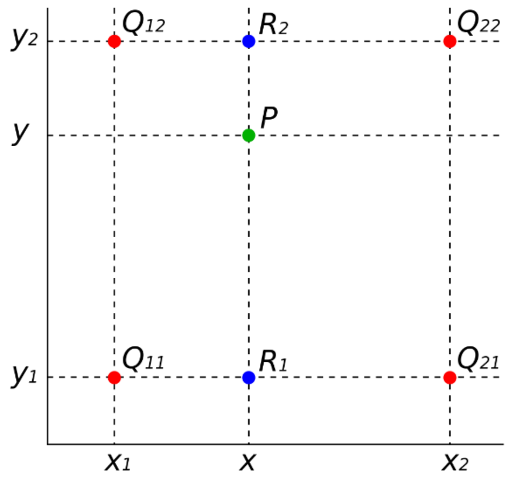
- For more detailed explanation refer to the link below

$$f(x, y_1) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21})$$

$$f(x, y_2) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22})$$

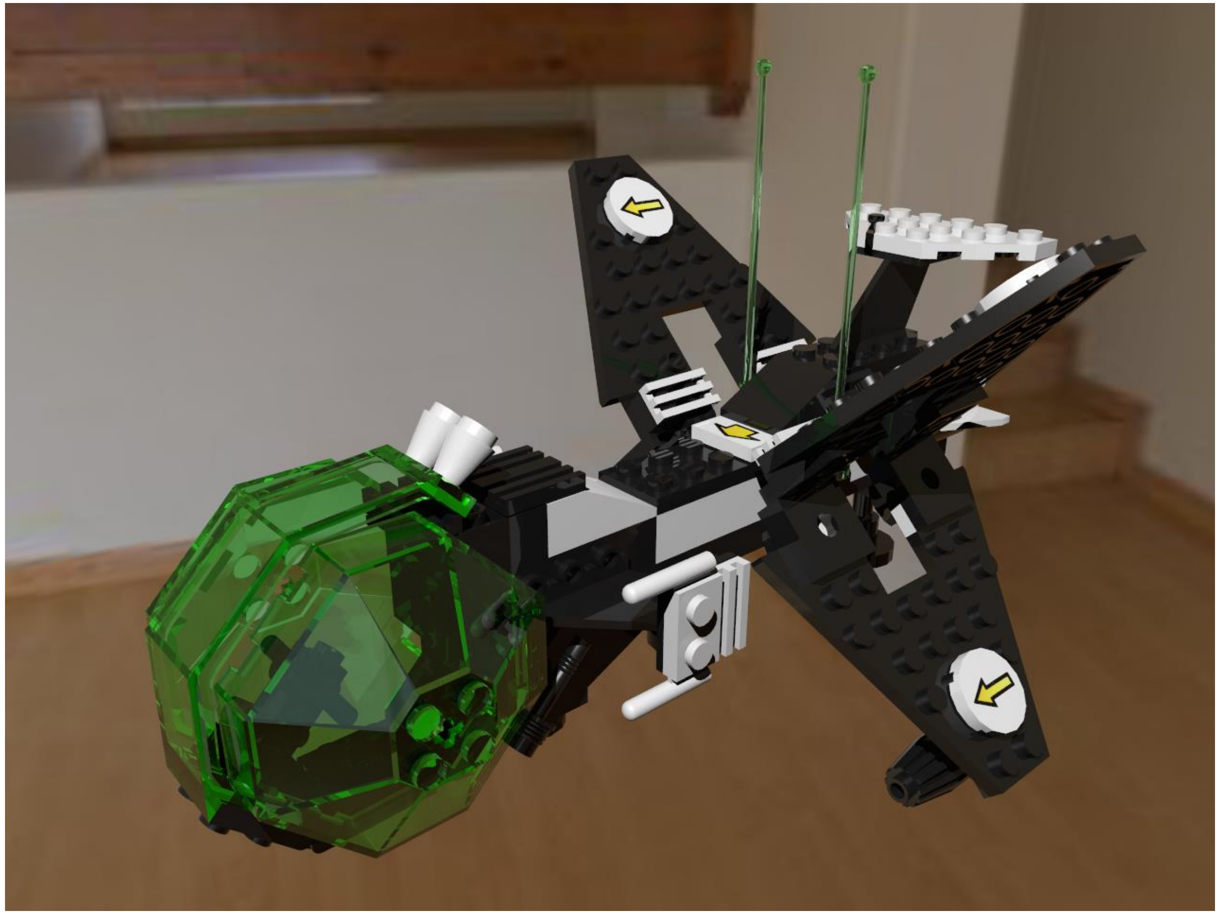
$$\begin{aligned}\boxed{f(x, y)} &\approx \frac{y_2 - y}{y_2 - y_1} f(x, y_1) + \frac{y - y_1}{y_2 - y_1} f(x, y_2) \\ &= \frac{y_2 - y}{y_2 - y_1}\left(\frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21})\right) + \frac{y - y_1}{y_2 - y_1}\left(\frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22})\right) \\ &= \frac{1}{(x_2 - x_1)(y_2 - y_1)}\left(f(Q_{11})(x_2 - x)(y_2 - y) + f(Q_{21})(x - x_1)(y_2 - y) + f(Q_{12})(x_2 - x)(y - y_1) + f(Q_{22})(x - x_1)(y - y_1)\right)\end{aligned}$$

This is the resulting value of interpolated quantity at the point P



Source: https://en.wikipedia.org/wiki/Bilinear_interpolation

Excercise



- In this scene, everything is made of transparent or opaque dielectric material (e.g. glass, plastic, wood, pure water, wax, rubber etc.), and single layered
- Dielectric do not tint specular reflection with objects color
- IOR of plastics is ~ 1.460, IOR of glass is ~ 1.5
- Reflection coefficient of reflected ray is estimated by Schlick's approximation

Composition

- In the case of the image from the previous slide, the color $\boldsymbol{C}^i$ from i-th hit point on opaque dielectric surface is computed as follows

$$\boldsymbol{C}^i = \boldsymbol{C}_{Phong} + \boldsymbol{C}_{refl}\, R(\theta_i)$$

where

Color of the light source, e.g. $(1, 1, 1)$

$$\boldsymbol{C}_{Phong} = \boldsymbol{l}_c \left(\boldsymbol{m}_a + \vartheta(\boldsymbol{p}, \boldsymbol{l}_p) \left(\boldsymbol{m}_d (\hat{\boldsymbol{l}}_d \cdot \hat{\boldsymbol{n}}) + \boldsymbol{m}_s (\hat{\boldsymbol{l}}_r \cdot \hat{\boldsymbol{v}})^{\gamma} \right) \right),$$

Visibility function between the hit point $\boldsymbol{p}$ and the position $\boldsymbol{l}_p$ of an omni light

$\boldsymbol{C}_{refl}$ is color returned by the reflected ray, and R is Schlick's approximation of hit point reflectivity

Composition

6887_allied_avenger.mtl

```
newmtl white_phong
  Ns 20
  Ni 1.460
  Ka 0.01 0.01 0.01
  Kd 0.95 0.95 0.95
  Ks 0.8 0.8 0.8
  shader 3

newmtl black_phong
  Ns 20
  Ni 1.460
  Ka 0.01 0.01 0.01
  Kd 0.1 0.1 0.1
  Ks 0.8 0.8 0.8
  shader 3

newmtl white_phong_4150p04
  Ns 20
  Ni 1.460
  Ka 0.01 0.01 0.01
  Kd 0.95 0.95 0.95
  Ks 0.8 0.8 0.8
  map_Kd 4150p04.jpg
  shader 3

newmtl white_phong_3069bp13
  Ns 20
  Ni 1.460
  Ka 0.01 0.01 0.01
  Kd 0.95 0.95 0.95
  Ks 0.8 0.8 0.8
  map_Kd 3069bp13.jpg
  shader 3

newmtl green_glass
  Tf 0.4 0.001 0.4
  Ni 1.5
  shader 4
```

Notes:

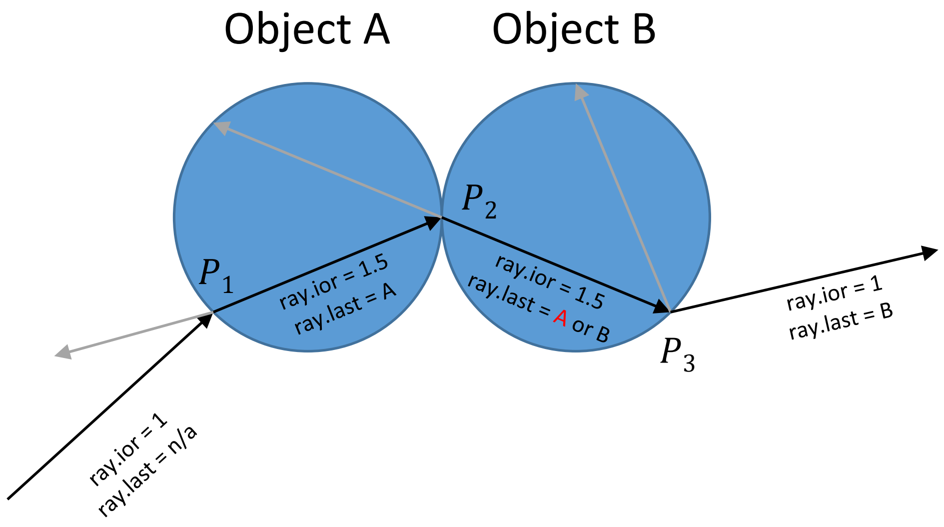
Ka, *Kd*, and *Ks* are treated as RGB values stored in sRGB gamma compressed space to match values stored in texture files

Tf (in case of shader 4 - glass) is treated as RGB value representing attenuation coefficient

Recursion depth is set to 10

Problematic Cases

- Two objects have one or more contact points



Expected order of indices at three hit points
P_1: n_1=1, n_2=1.5
P_2: n_1=1.5, n_2=1.5
P_3: n_1=1.5, n_2=1

List of visited interfaces
Air→ Glass, Glass → Glass, Glass→Air

There is no way how to determine the correct indices for P_2 case without any further assumptions

~~Assumption 1: Objects are closed and non-self penetrating~~
~~Assumption 2: Ray remembers current environment ior and id of last visited interface;~~